Week 8:

Magnetostatics

Magnetism

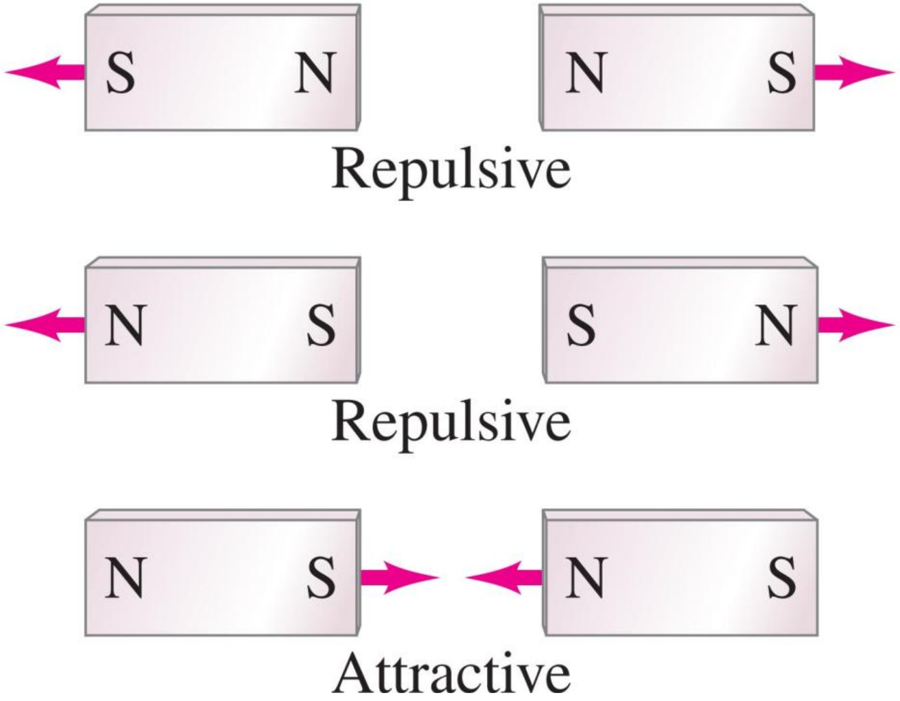
Greeks were probably the first, who recognized magnetism 2.5 thousand years ago. They noticed that natural mineral **lodestone** (magnetite, Fe_3O_4) **attracts** or **repel** **the same stones** and iron. This minerals could be magnetized by lighting bolts and are capable to stay magnetized for long (not like iron).

Similar to charges, magnetized bodies do interact, but they are **NOT** charged

Magnets

Magnets have two ends – <u>poles</u> – called north and south.

Like poles repel; unlike poles attract.



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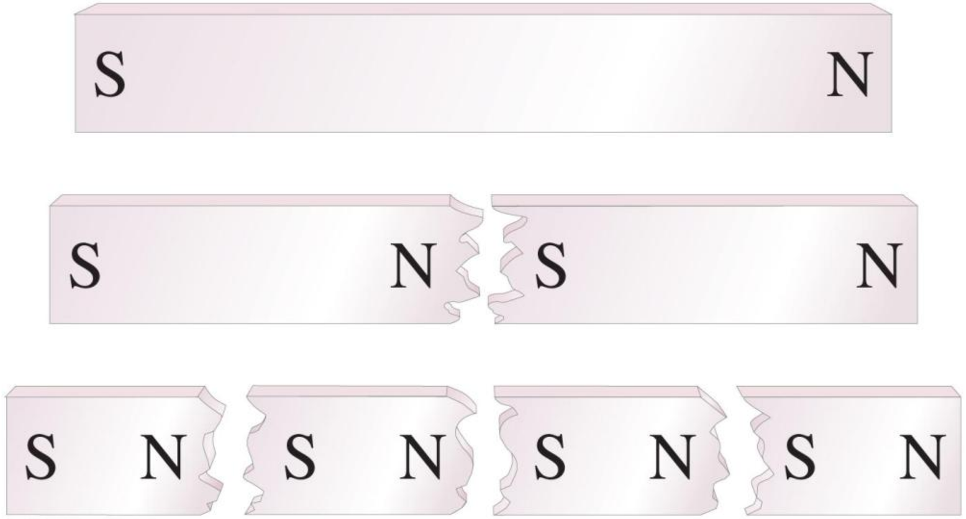
https://auditoires-physique.epfl.ch/experiment/729/action-dun-conducteur-sur-une-boussole-exp-doersted

Magnets

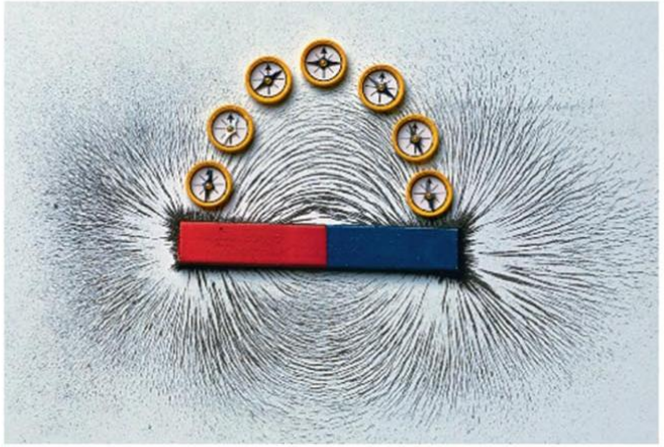
However, if you cut a magnet in half, you don't get a north pole and a south pole – you get two smaller magnets.



Unlike electric charges, there are NO single magnetic charges (monopoles).

Magnets and Magnetic Fields

Small magnetic particles arrange between two poles of a magnet along lines, called *Lines of Magnetic field*. Different from Electric field, lines of Magnetic field always make closed loops (in *N* to *S* direction)



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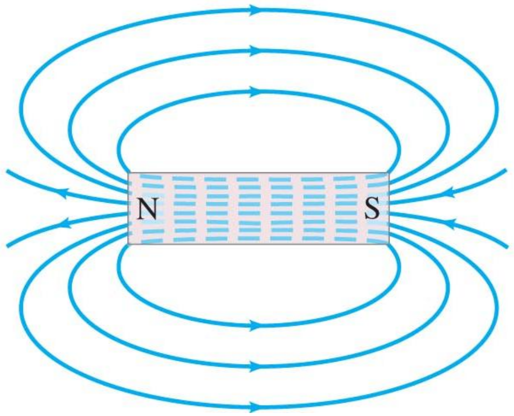
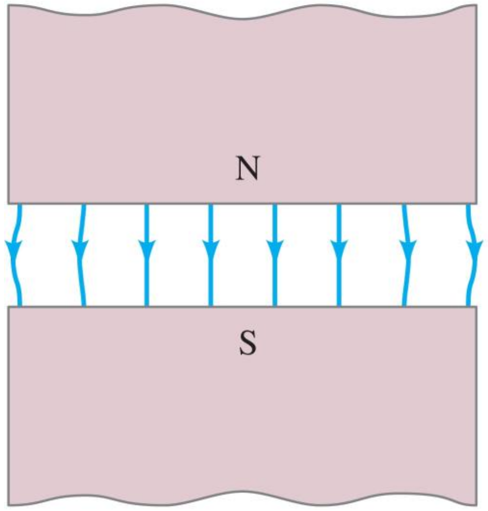
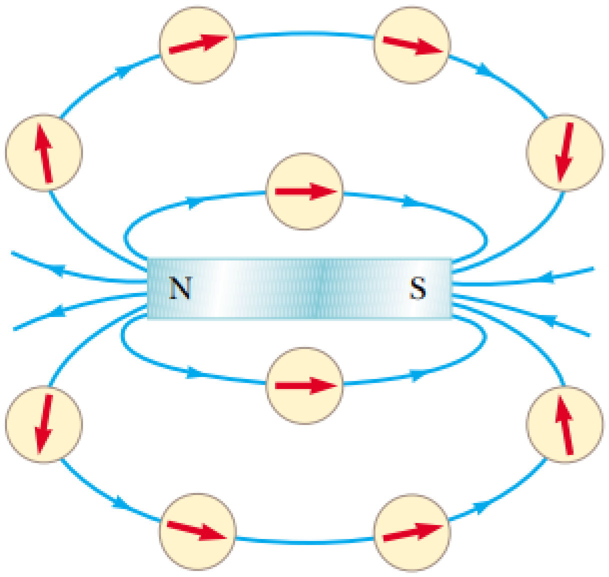


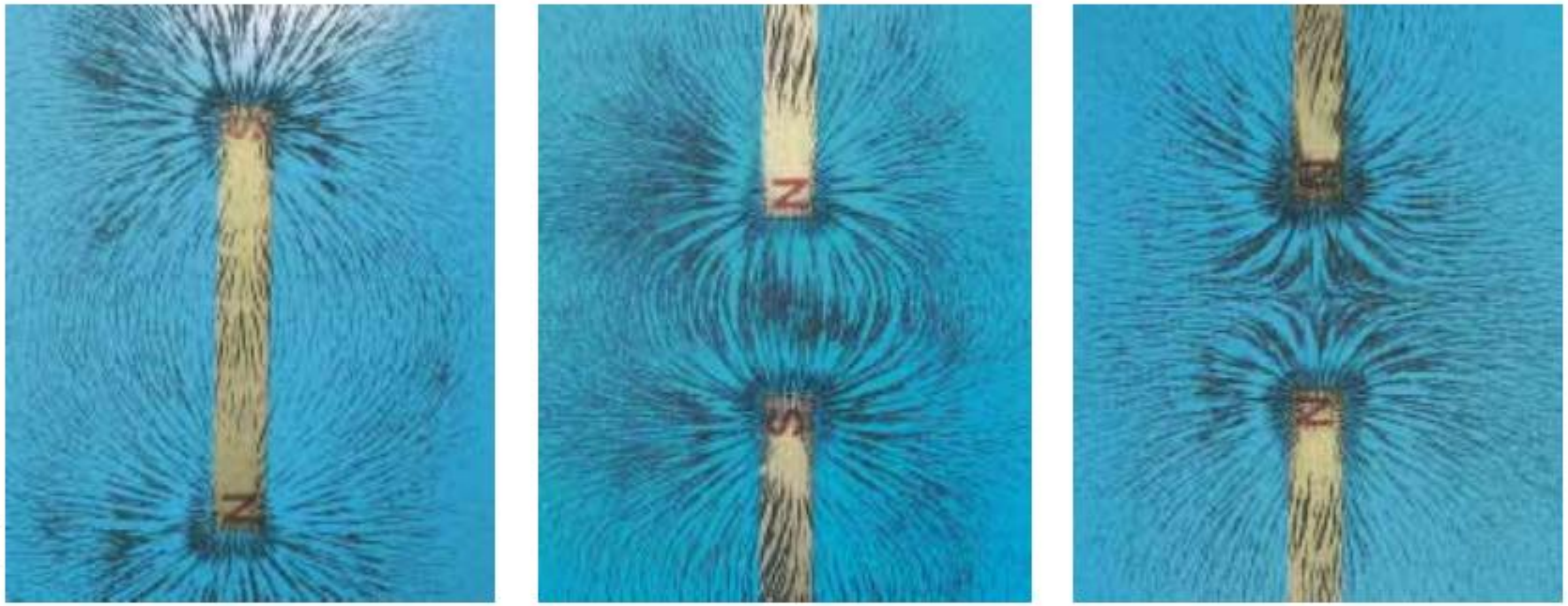


Magnetic field lines

The magnetic field lines *of a bar magnet can be traced with the aid of a compass. Note that the magnetic field lines outside the magnet point* ***away from north*** *poles and* ***toward south poles****. One can display magnetic field patterns of a bar magnet using small iron filings.*



Compass needles can be used to trace the magnetic field lines in the region outside a bar magnet.



(a) (b) (c)

(a) Magnetic field pattern surrounding a bar magnet as displayed with iron filings.
(b) Magnetic field pattern between opposite poles (N–S) of two bar magnets.
(c) Magnetic field pattern between like poles (N–N) of two bar magnets.

Earth's Magnetic Field

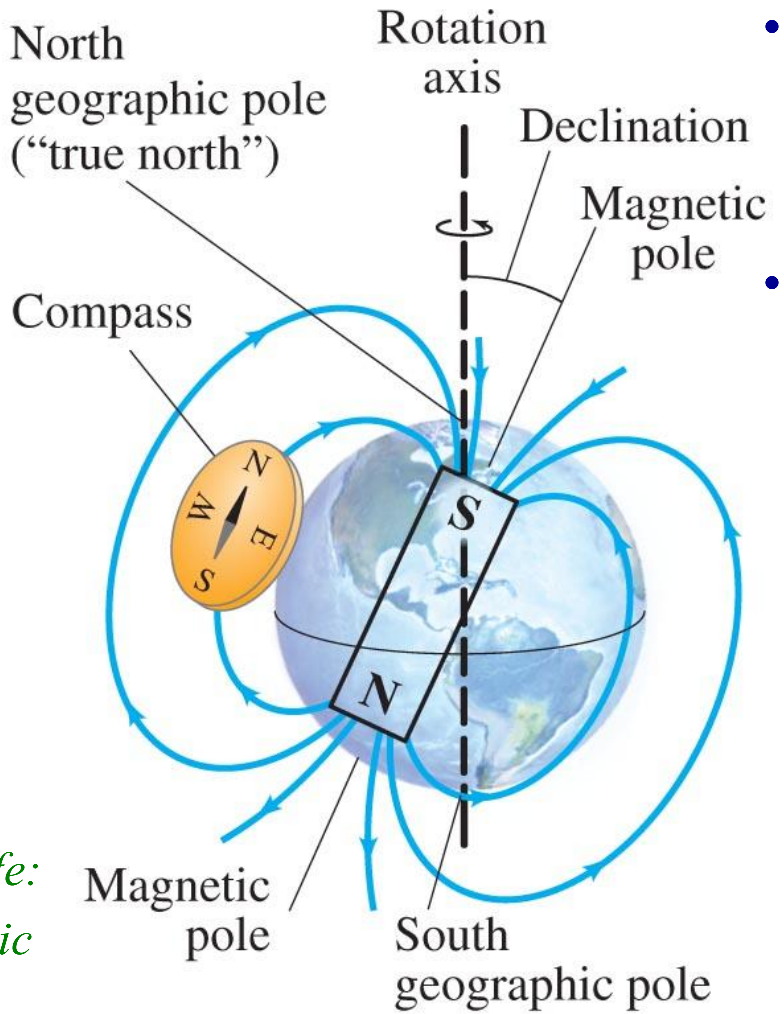
The Earth's magnetic field is similar to that of a bar magnet.

Compass has a magnetized needle that aligns **S**-**N**, helping in navigations since many centuries

Note that the Earth's "North Pole" is really a south magnetic pole, as the north ends of magnets are attracted to it.

The Van Allen belts are made up of charged particles trapped by the Earth's nonuniform magnetic field.

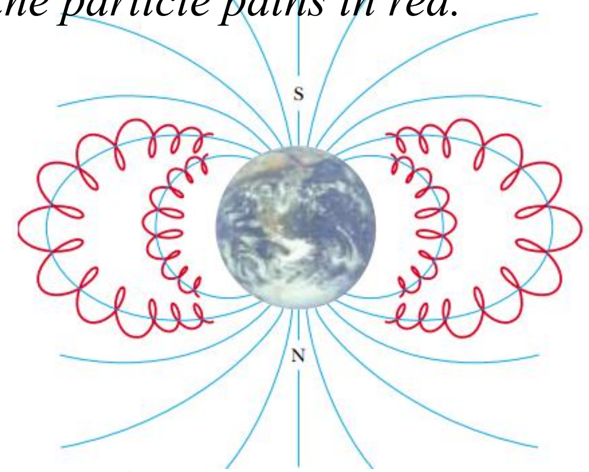
The Earth's magnetic field is vital for life: it protects all living from highly energetic ionic cosmic particles



Where does the Earth's magnetic field come from?

- **The Earth has a core of iron (2400 km in diameter), and it can be a giant magnet, *but*…**
- **From electric currents circulating in the molten material in the Earth's outer core ?**

The magnetic field lines are in blue and the particle paths in red.

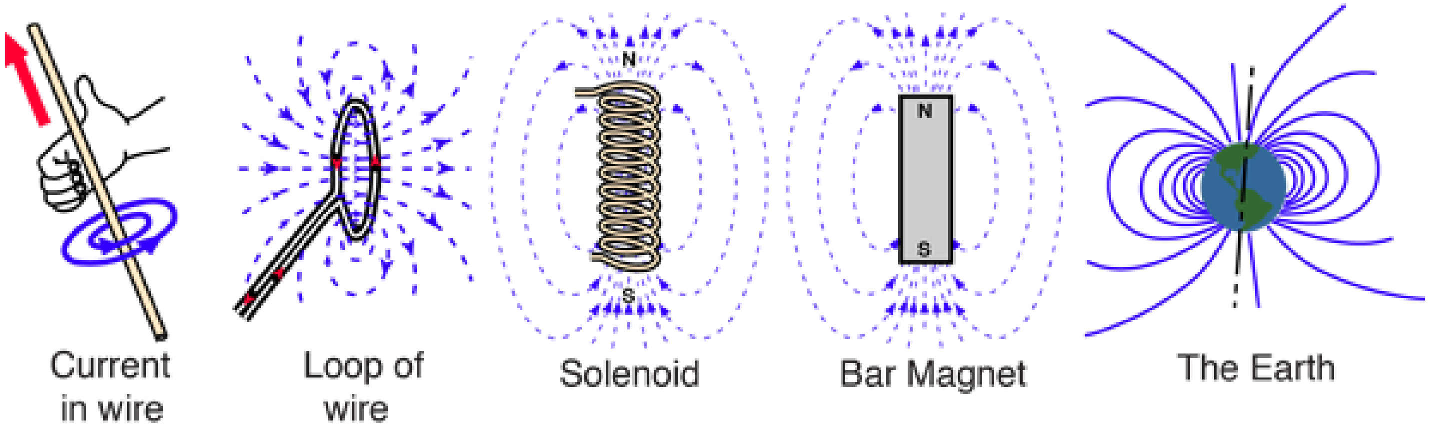


***N* and *S* reverse about every 300'000 years…**

Sources of the magnetic field

Movement of charges

Intrinsic magnetic moments of particles (electrons, protons, neutrons...)

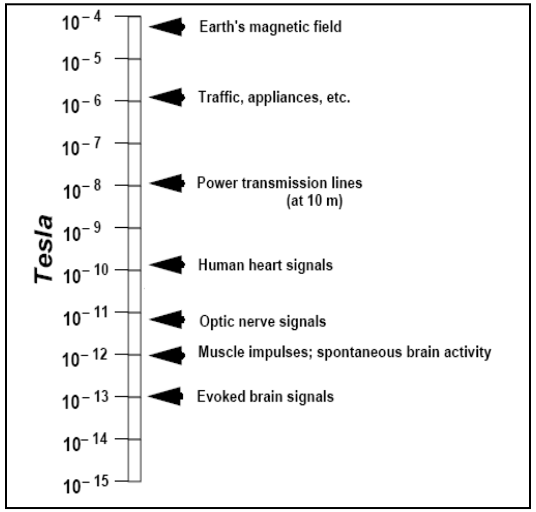


Coils with current: < 100 T typ.

permanent magnet: < 1 T typ.

Earth (on the surface): ~ 10 mT

Sources of "weak"magnetic field

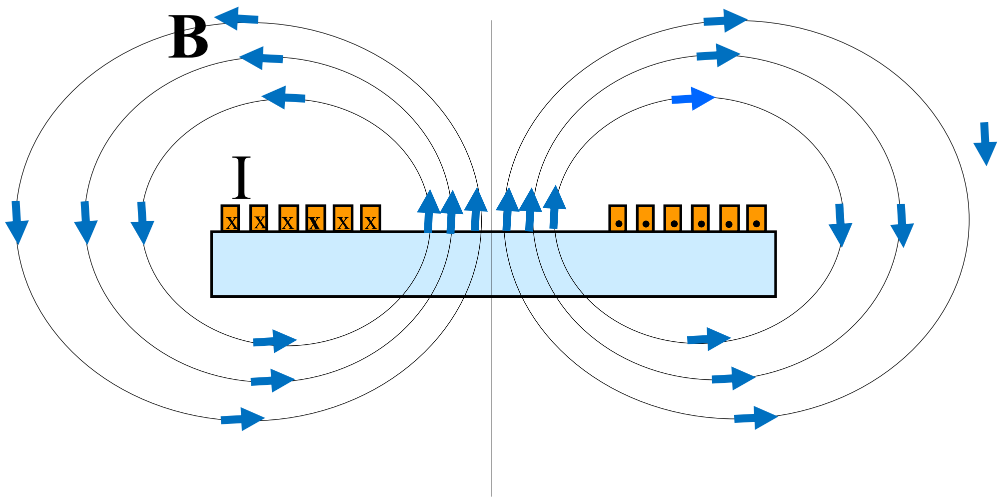


A distribution of **static electrical charges** produces a **static electric field**.

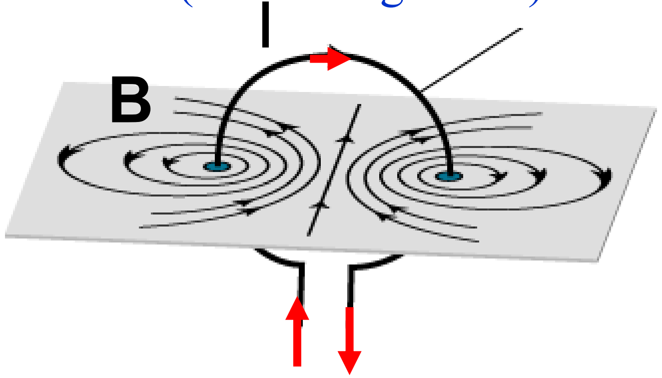
A distribution of **stationary electrical currents** produces a **static magnetic field**.

What we will see (next lecture): Magnetic field produced by different "structures"

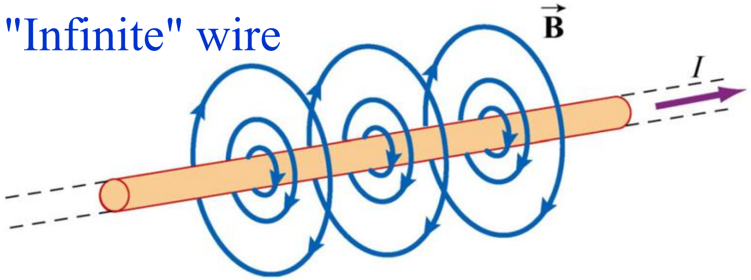
Planar coil (with multiple turns)



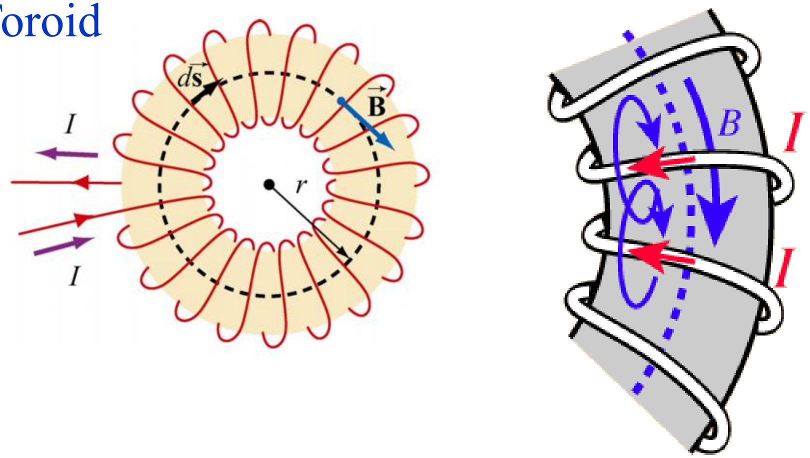
Planar coil (with a single turn)



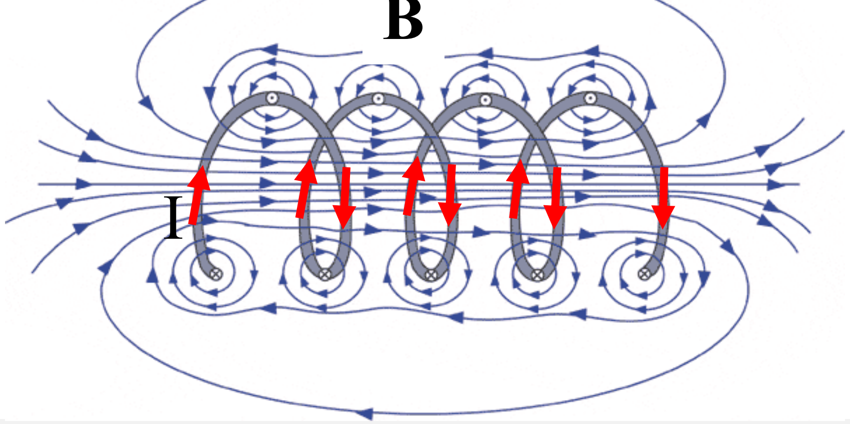
"Infinite" wire



Toroid

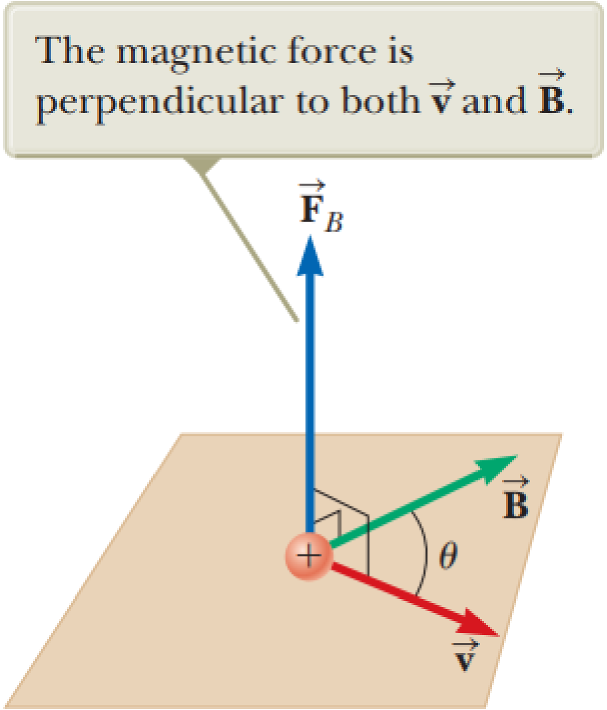


Solenoid



Summary of observations

- Magnetic interactions relate to magnetic field vector $\vec{B}$
- Superposition principle is applicable to magnetic field !
- Magnetic field acts on *moving* charges (e.g., on current)
- Magnetic *Force* increases proportionally to
  - magnetic field
  - charge
  - charge velocity,
  - but depends on orientation of the field $\vec{B}$ and velocity $\vec{v}$.



$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$F_B = |q| vB \sin\theta$$

We can regard this equation as an **operational definition** of the **magnetic field** at some point in space. That is, **the magnetic field is defined in terms of the force acting on a moving charged particle.**

unit of magnetic field is TESLA

$$1\ \mathrm{T} = 1\ \frac{\mathrm{N}}{\mathrm{C \cdot m/s}} \qquad 1\ \mathrm{T} = 1\ \frac{\mathrm{N}}{\mathrm{A \cdot m}}$$

Magnetic field doesn't change kinetic energy of charge!

Magnetic Fields and Forces

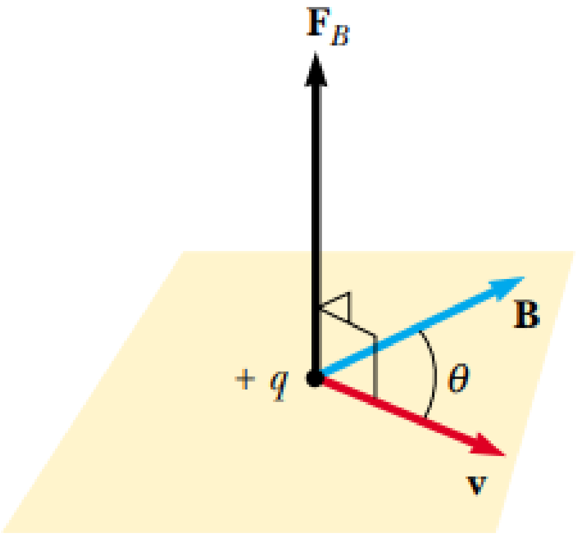
- *When the particle's velocity vector makes any angle $\theta \neq 0$ with the magnetic field, **the magnetic force acts in a direction perpendicular to both v and B**; that is, $\boldsymbol{F_B}$ is perpendicular to the plane formed by **v** and **B** (Fig. a).*

- *The magnetic force $\boldsymbol{F_B}$ exerted on a positive charge is in the direction opposite to the direction of the magnetic force exerted on a negative charge moving in the same direction (Fig. b).*

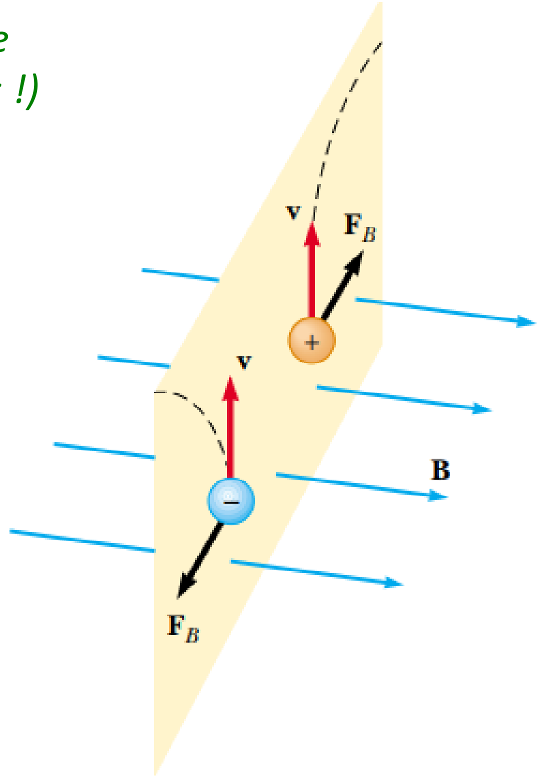
$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$$

Vector expression for the magnetic force on a charged particle moving in a magnetic field

(conventionally, current direction is the direction of motion of positive charges !)



(a)



(b)

*The direction of the magnetic force $\boldsymbol{F_B}$ acting on a charged particle moving with a velocity **v** in the presence of a magnetic field **B**. (a) The magnetic force is perpendicular to both **v** and **B**. (b) Oppositely directed magnetic forces $\boldsymbol{F_B}$ are exerted on two oppositely charged particles moving at the same velocity in a magnetic field. The dashed lines show the paths of the particles.*

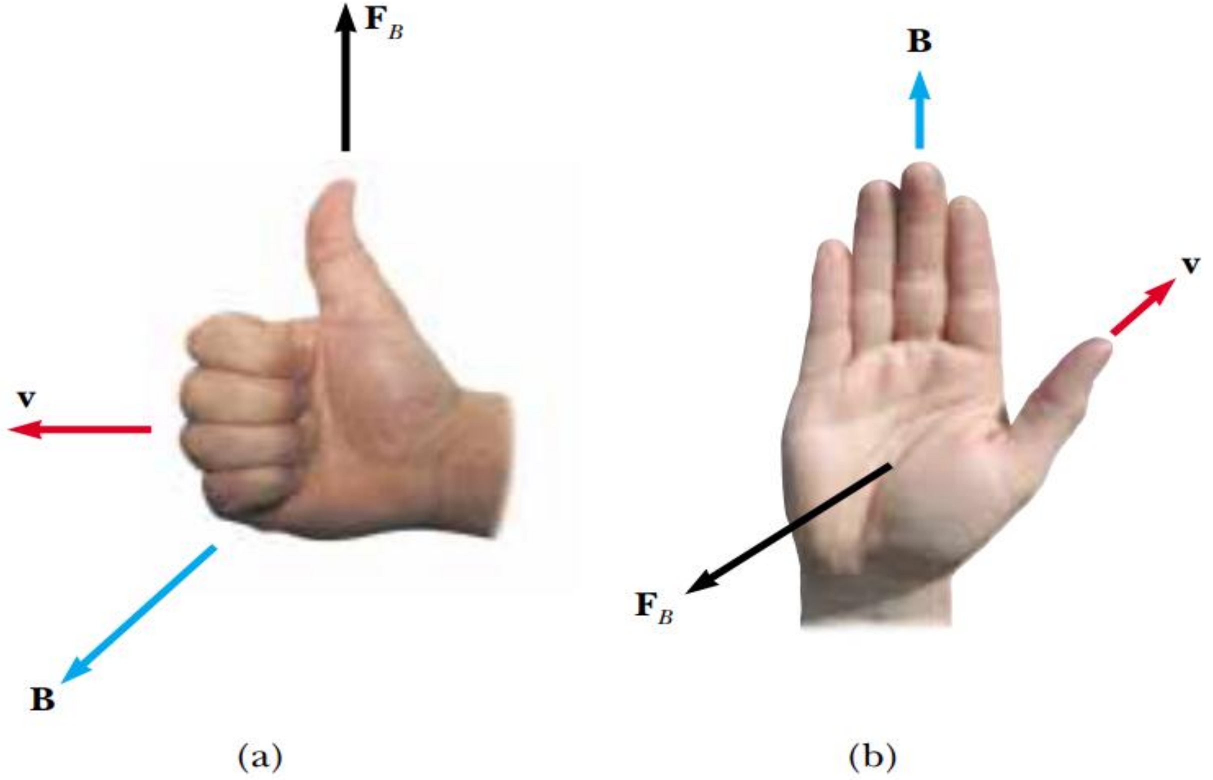
right-hand rules for determining the direction of the force

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$$

The magnitude of the magnetic force on a charged particle is

$$F_B = |q|vB\sin\theta$$

where θ is the smaller angle between v and B. From this expression, we see that $\boldsymbol{F_B}$ *is zero when* ***v*** *is parallel or antiparallel to* ***B*** *(θ =0 or θ =180°) and maximum when* ***v*** *is perpendicular to* ***B*** *(θ = 90°).*



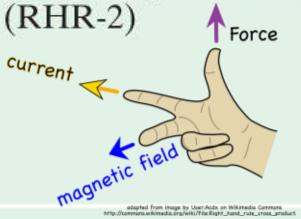
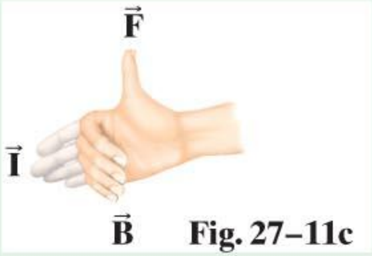
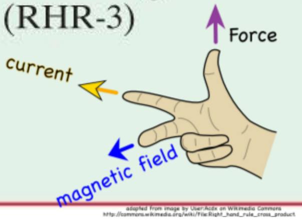
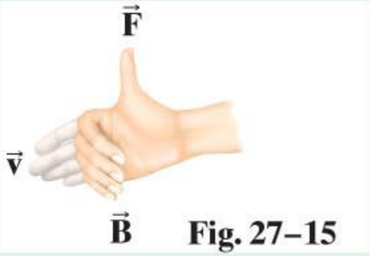
Two implementations of the ***right-hand rules*** *for determining the direction of the magnetic force* $\boldsymbol{F_B} = q\boldsymbol{v}\mathrm{x}\boldsymbol{B}$ *acting on a particle with charge q moving with a velocity v in a magnetic field B.*

(a) In this rule, the fingers point in the direction of v, with B coming out of your palm, so that you can curl your fingers in the direction of B. The direction of $\boldsymbol{v}\mathrm{x}\boldsymbol{B}$*, and the force on a positive charge, is the direction in which the thumb points.*

(b) In this rule, the vector v is in the direction of your thumb and B in the direction of your fingers. The force $\boldsymbol{F_B}$ *on a positive charge is in the direction of your palm, as if you are pushing the particle with your hand.*

Force on an Electric Charge Moving in a Magnetic Field

TABLE 27–1 Summary of Right-hand Rules (= RHR)

Physical Situation	Example	How to Orient Right Hand	Result
1. Magnetic field produced by current (RHR-1) 	 Fig. 27–8c	Wrap fingers around wire with thumb pointing in direction of current I	Fingers point in direction of $\vec{\mathbf{B}}$
2. Force on electric current I due to magnetic field (RHR-2) 	 Fig. 27–11c	Fingers point straight along current I, then bend along magnetic field $\vec{\mathbf{B}}$	Thumb points in direction of the force $\vec{\mathbf{F}}$
3. Force on electric charge $+q$ due to magnetic field (RHR-3) 	 Fig. 27–15	Fingers point along particle's velocity $\vec{\mathbf{v}}$, then along $\vec{\mathbf{B}}$	Thumb points in direction of the force $\vec{\mathbf{F}}$

Magnetic *vs* Electric Forces

Electric force acts **in the direction of the electric field.**

$$\vec{F}_E = q\vec{E}$$

Electric force is nonzero even if v=0.

Magnetic force acts **perpendicular to the magnetic field.**

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

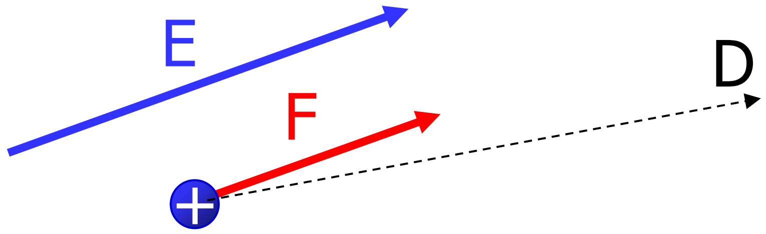
Magnetic force is zero if v=0.

$$\vec{F}_B(v=0) = (q)\left(\vec{0}\right) \times \vec{B} = 0$$

Magnetic and Electric Forces

Electric force does work in displacing a charged particle.

$\vec{F}_E = q\vec{E}$

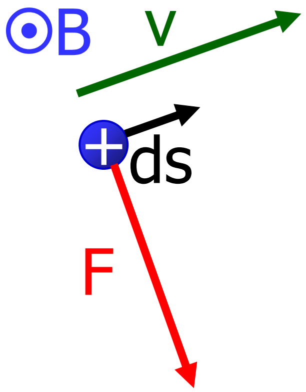


$W_F = \vec{F} \cdot \vec{D}$

Electric force can be used to accelerate particles.

Magnetic force does not do work in displacing a charged particle!

$\vec{F}_B = q\vec{v} \times \vec{B}$



$W_F = \vec{F} \cdot d\vec{s} = \mathbf{0}$

Important differences between electric and magnetic forces

- *The electric force acts along the direction of the electric field, whereas the magnetic force acts perpendicular to the magnetic field.*

- *The electric force acts on a charged particle regardless of whether the particle is moving, whereas the magnetic force acts on a charged particle only when the particle is in motion.*

- *The electric force does work in displacing a charged particle, whereas the magnetic force associated with a steady magnetic field does no work when a particle is displaced because the force is perpendicular to the displacement.*

*On the basis of the work–kinetic energy theorem, **we conclude that the kinetic energy of a charged particle moving through a magnetic field cannot be altered by the magnetic field alone**.*
In other words, when a charged particle moves with a velocity v through a magnetic field B, the field can alter the direction of the velocity vector but cannot change its magnitude or kinetic energy of the particle.

Trajectory of a charged particle in a field B

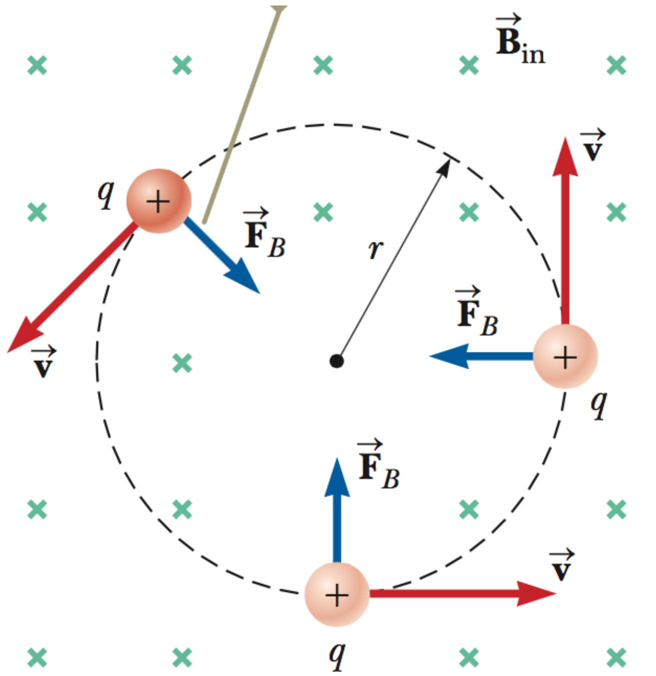
Uniform B field and velocity v⊥B

- magnetic field is homogeneous and directed away
- Charge velocity is orthogonal to MF

What is the charge trajectory?

When a force is always orthogonal to the velocity, the latter doesn't change its magnitude, but only its direction.
The trajectory of a body then is a circle:

Uniform circular motion



Centripetal Force

$$F_c = \frac{mv^2}{r}; \Longrightarrow r = \frac{mv^2}{F_c}; \quad \omega_c = \frac{v}{r} = \frac{F_c}{mv}$$

$$F_c = F_B = qvB \Longrightarrow r = \frac{mv^2}{qvB} = \boxed{\frac{mv}{qB}}$$

In our case

Larmor radius

Cyclotron cyclic frequency

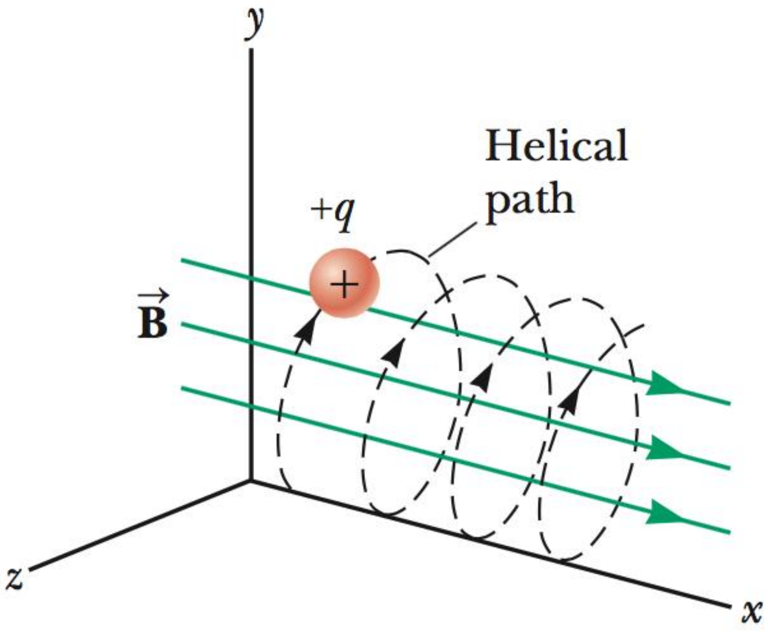
$$\omega_c = \frac{qvB}{mv} = \boxed{\frac{q}{m}B}$$

Doesn't depend on the velocity v

Motion of a charge in magnetic field

When the charge velocity vector is not orthogonal to magnetic field, only the velocity component $v_\perp$ contributes to cyclotron motion,

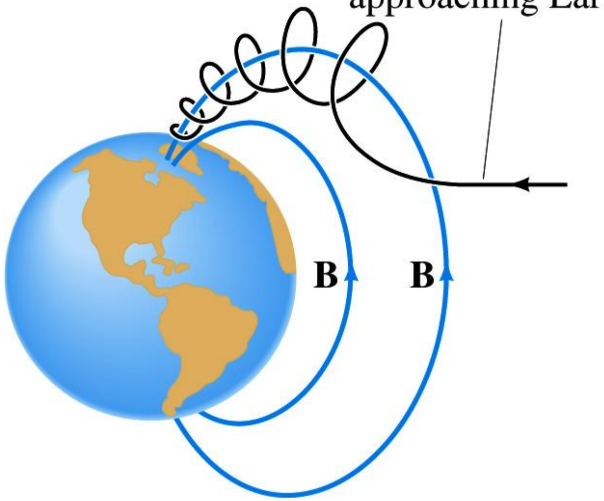
$$r = \frac{mv_\perp}{qB} \qquad \omega_c = \frac{q}{m}B$$



while the $v_{\parallel}$ component along the field doesn't change.
The trajectory of the charge then is a helix.

- In a uniform and constant magnetic field ω_c depends only on q/m;
- This fact is used in Ion Cyclotron Resonance mass spectrometers (ICR MS) to determine m/q of ions by measuring ω_c.

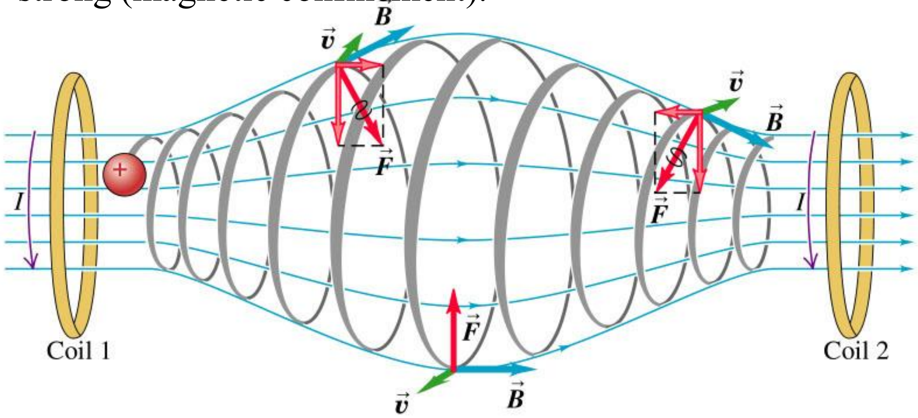
1. Charged particles approaching each other of the earth)

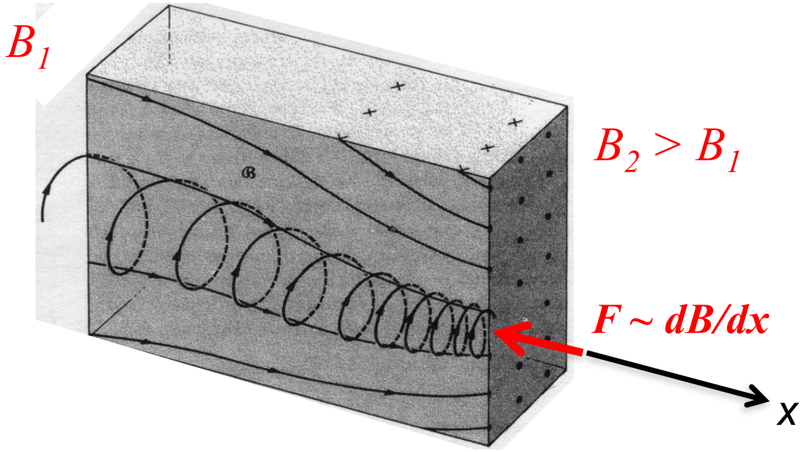


Electrons and protons from the Sun are trapped by the Earth's magnetic field. A charged particle spirals between two magnetic mirrors near the North and South poles. These particles collide with atoms and molecules in the atmosphere. The de-energization of these atoms and molecules creates the Aurora.

2. Magnetic "mirror"

The mirror effect results from the tendency for charged particles to bounce back from the region where the field is strong (magnetic confinement).

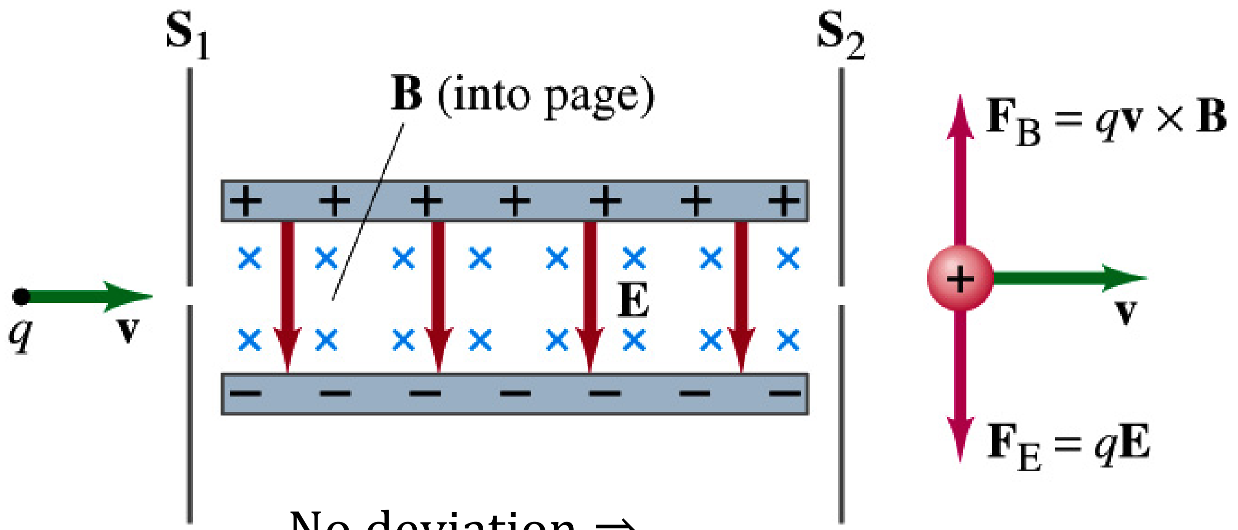




$$r_L = \frac{mv_\perp}{qB}$$

Lorentz Force

3. Velocity selector



No deviation ⇒
$\mathbf{F}_B = \mathbf{F}_E \Rightarrow qvB = qE \Rightarrow v = E/B$

Particles passing the center of S_2
(*i.e. the particles selector*") have velocity:

$\boldsymbol{v} = \dfrac{\boldsymbol{E}}{\boldsymbol{B}}$ *Velocity selector*

The force on an electric charge moving in an electromagnetic field is called the Lorentz force; its complete form is:

$$\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$$

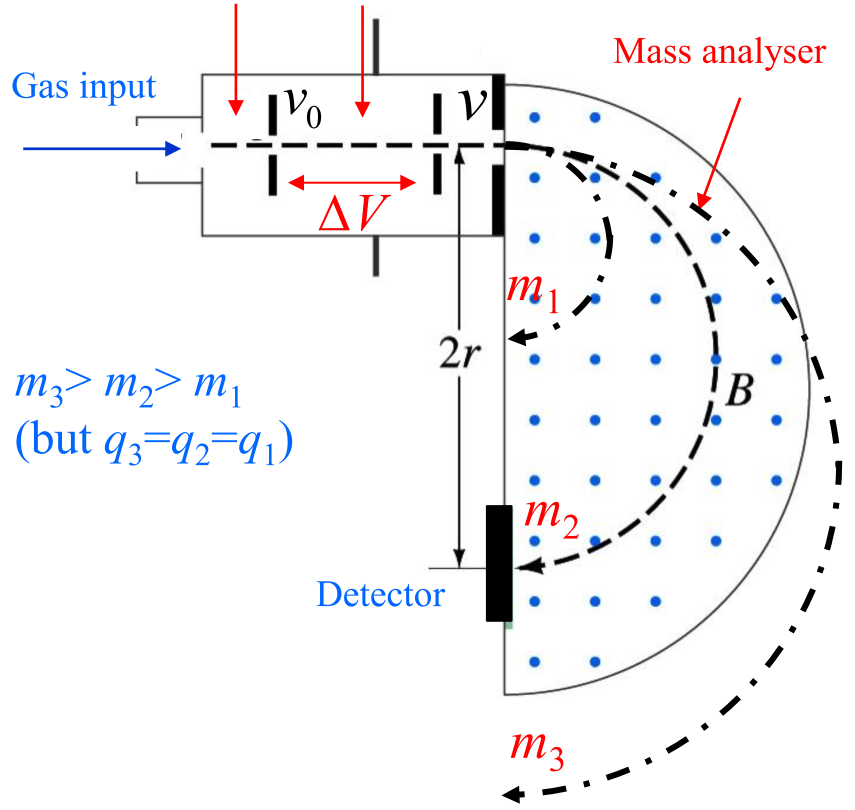
Under the condition of orthogonality of $\vec{v}, \vec{E}$ and $\vec{B}$ we can have $\boldsymbol{E = vB}$:
the two forces may compensate each other, such that a charge doesn't experience any net force.

DEMO

https://auditoires-physique.epfl.ch/experiment/415/deviation-electrostatique-et-magnetique-dun-faisceau-electronique

4. mass spectrometer

It consists of 3 sections:

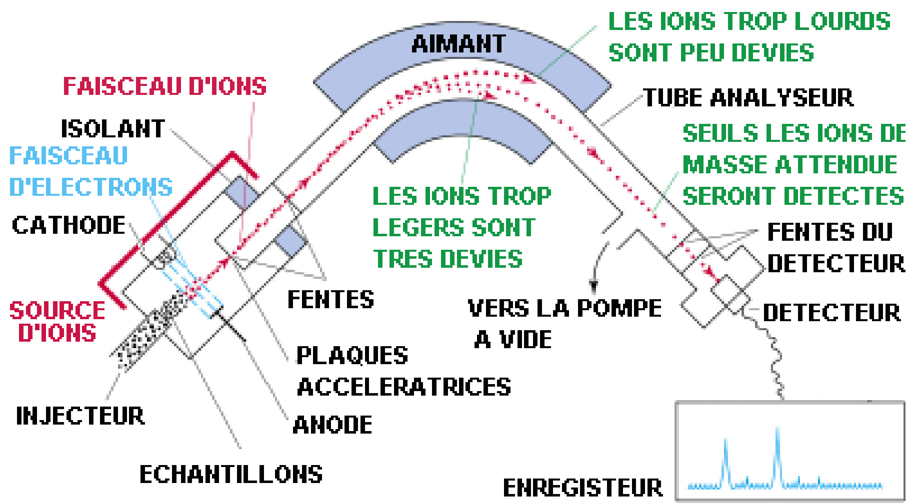


$m_3 > m_2 > m_1$
(but $q_3 = q_2 = q_1$)

Ionisation: atoms → ions
(by bombardment with an electron beam)

Acceleration: $v_0 \cong 0; \quad \frac{1}{2} m v^2 = q \Delta V$;

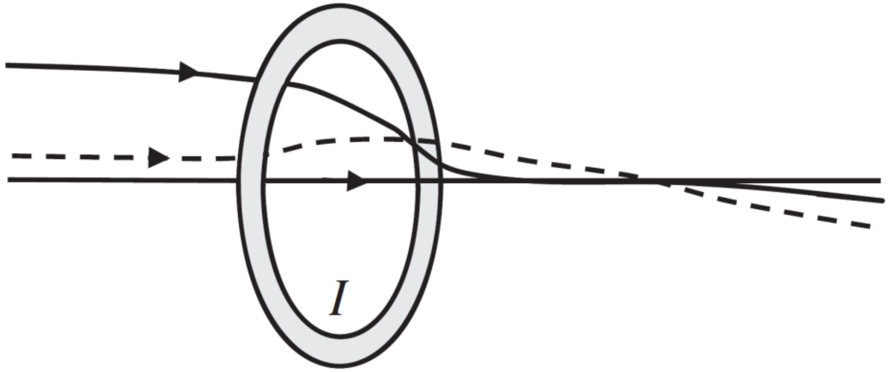
Mass analyser: $r = \frac{mv}{qB} = \frac{1}{B} \left(\frac{2m}{q} \Delta V \right)^{1/2}$



$$\frac{\mathbf{m}}{\mathbf{q}} = \frac{\boldsymbol{rB}}{\boldsymbol{v}}$$

5. Magnetic lens for electron microscope

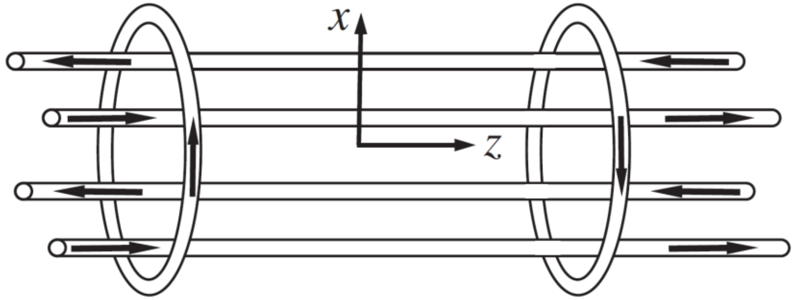
Charged particles (electrons) with initially parallel (or nearly parallel) trajectories are focused by a circular current loop.



6. Magnetic trapping

Local minimum trapping of the magnetic field occurs for atoms whose total angular momentum is anti-parallel to the local magnetic field.

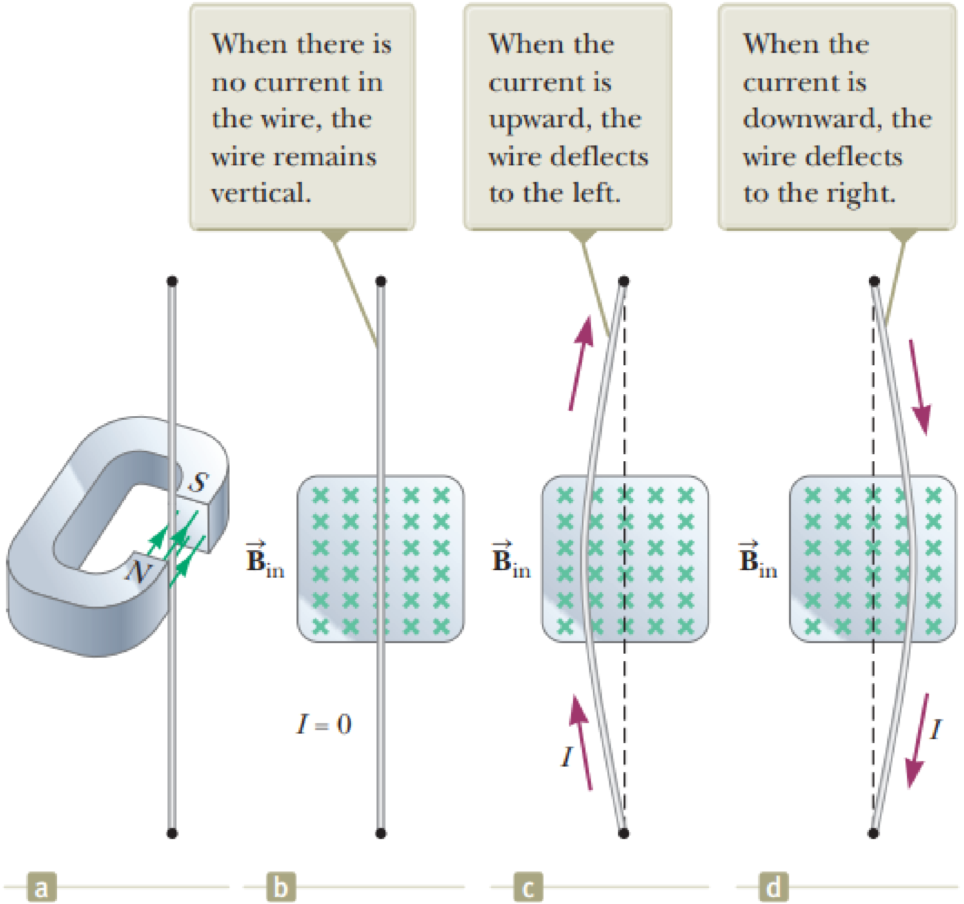
$$E_P = -\mathbf{m} \cdot \mathbf{B}$$

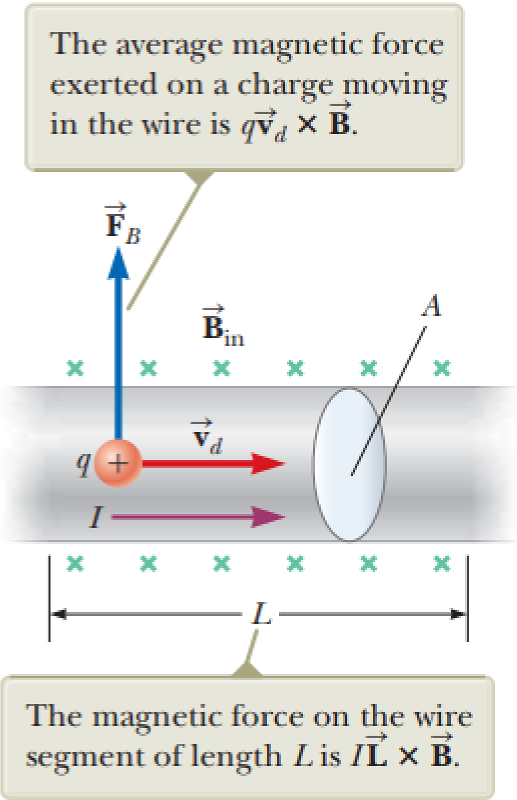


Note: It is impossible to produce a local maximum of the magnitude of the magnetic field in free space. This means that it is not possible to trap a particle with a magnetic moment parallel to the local magnetic field.

Figure 12.9: The Ioffe-Pritchard configuration produces a minimum of $|\mathbf{B}(\mathbf{r})|$ at its center. Arrows indicate the direction of current flow in each wire.

Magnetic force on a current in a wire





DEMO

https://auditoires-physique.epfl.ch/experiment/589/fil-dans-champ-magnetique-experience-de-faraday

$$\vec{F}_B = (q\vec{v}_d \times \vec{B})\,nAL$$

$$I = nqv_dA$$

n is the number of mobile charge carriers per unit volume

$$\boxed{\vec{F}_B = I\vec{L} \times \vec{B}}$$

$$d\vec{F}_B = I\,d\vec{s} \times \vec{B}$$

where **L** is a vector that points in the direction of the current *I* and has a magnitude equal to the length L of the segment.

This expression applies only to a straight segment of wire in a uniform magnetic field.

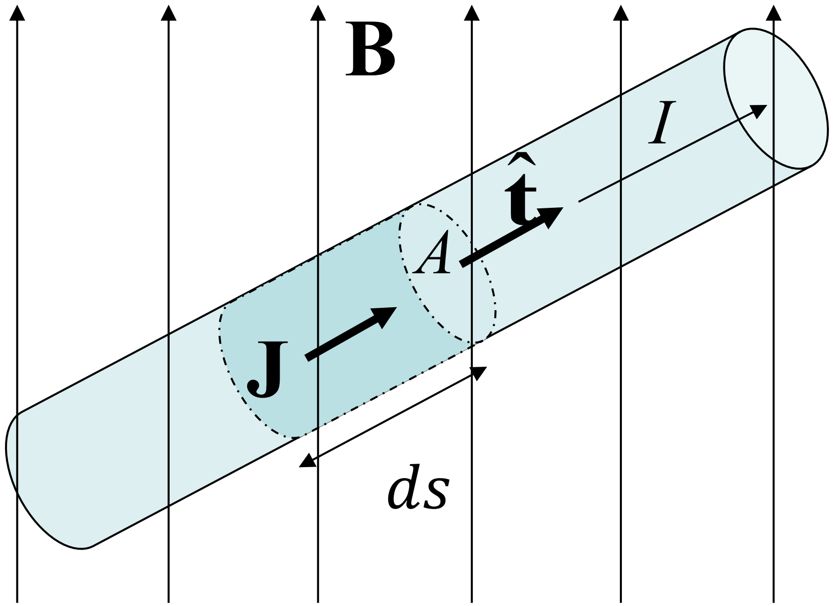
Magnetic force on a current-carrying conductor
in a field B

The force on a conductor is the Lorentz force on moving electric charges inside the conductor.

$$d\mathbf{s} = ds\hat{\mathbf{t}} \qquad \boldsymbol{A} = A\hat{\mathbf{t}}$$

Area of the section of the wire

$$\mathbf{J} = nq\mathbf{v} = nqv\hat{\mathbf{t}} \qquad I = \mathbf{J}\cdot\boldsymbol{A} = nqvA$$



Force on the volume ($dV = ds\,A$):

$$\sum_i (q_i\mathbf{v}_i \times \mathbf{B}) = (\mathbf{v}\times\mathbf{B})\sum_i q_i =$$
$$= (\mathbf{v}\times\mathbf{B})nqdsA = nq\mathbf{v}\times\mathbf{B}dsA = \mathbf{J}\times\mathbf{B}\,dsA$$

$\mathbf{F}_B$: Total force over a wire of length L

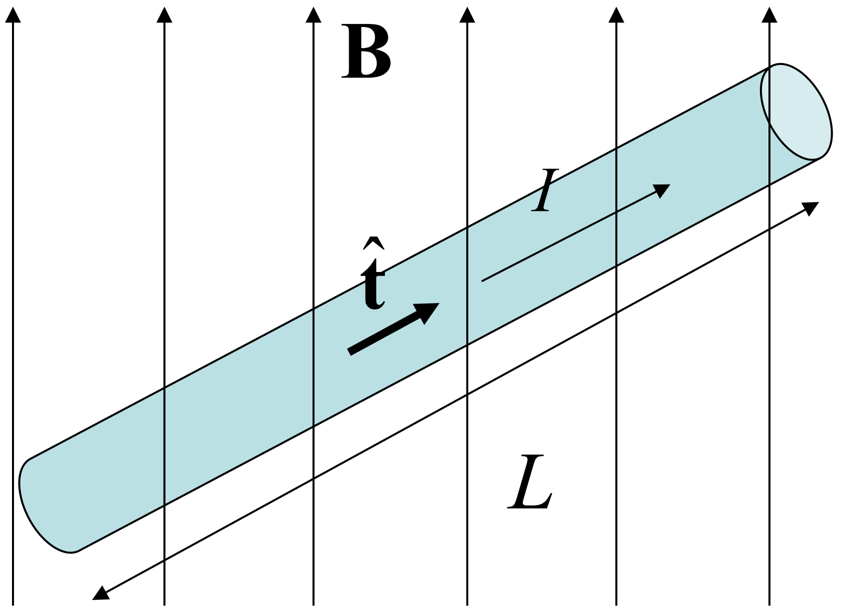
$\mathbf{f}$: Force per unit volume $dV = dsA$

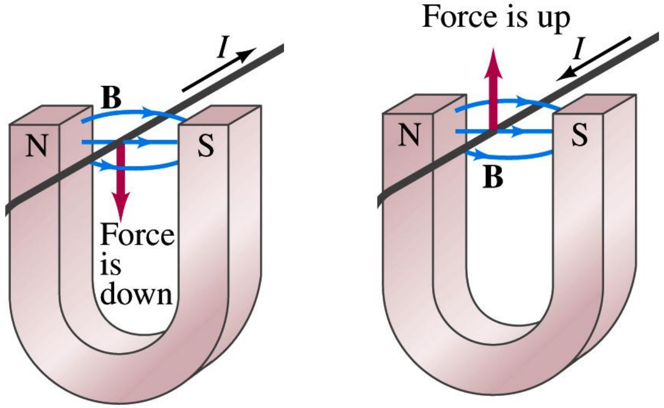
$$\mathbf{f} = nq\mathbf{v}\times\mathbf{B} = \mathbf{J}\times\mathbf{B} \quad \Rightarrow \quad \mathbf{F}_B = \int_V \mathbf{J}\times\mathbf{B}\,dV = I\int_L \hat{\mathbf{t}}\times\mathbf{B}\,ds = I\int_L d\mathbf{s}\times\mathbf{B}$$

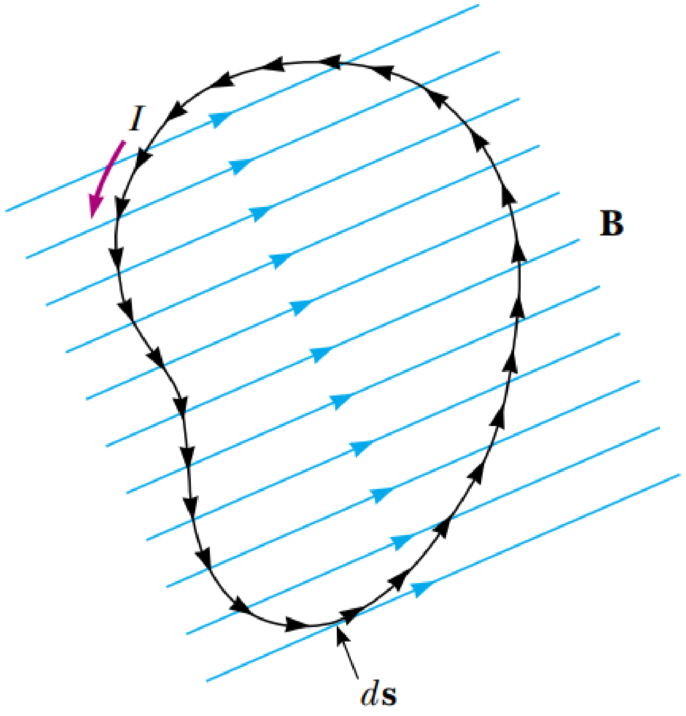
Exemple 1.: straight wire of length L in a field $\mathbf{B}$ uniform

$$\mathbf{F}_B = I\int_0^L d\mathbf{s} \times \mathbf{B} = I\mathbf{L}\times\mathbf{B}$$

$$\mathbf{L} = L\hat{\mathbf{t}}$$







Exemple 2.: Closed circuit (of any shape) in a field $\mathbf{B}$ uniform

$$\mathbf{F}_B = I\oint\hat{\mathbf{t}}\times\mathbf{B} = -I\mathbf{B}\times\oint d\mathbf{s} = 0$$

The resultant magnetic force on a closed circuit (of any shape) in a uniform **B** field is zero

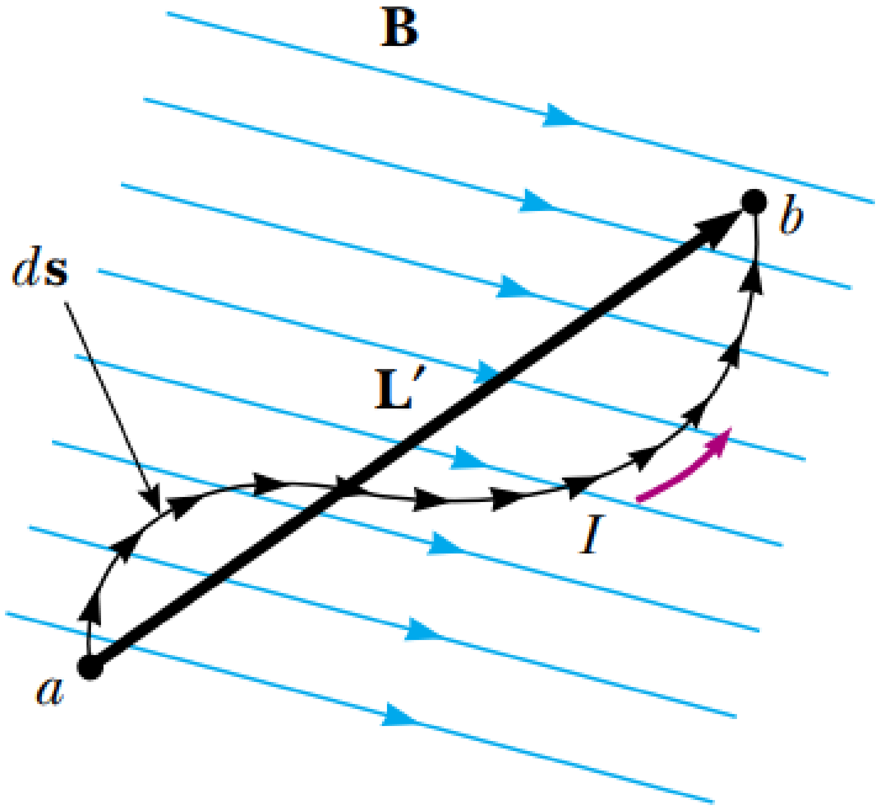
Exemple 3.: A curved wire carrying a current I (of any shape) and located in a field **B** uniform

Because the field is uniform, we can take B outside the integral, and we obtain

$$\mathbf{F}_B = I\int_a^b (d\mathbf{s} \times \mathbf{B}) = I\left(\int_a^b d\mathbf{s}\right) \times \mathbf{B}$$

But the quantity $\int_a^b d\mathbf{s}$ *represents the vector sum of all the length elements* $d\mathbf{s}$ *from a to b. From the law of vector addition, the sum equals the vector* $\mathbf{L}'$*, directed from a to b.*

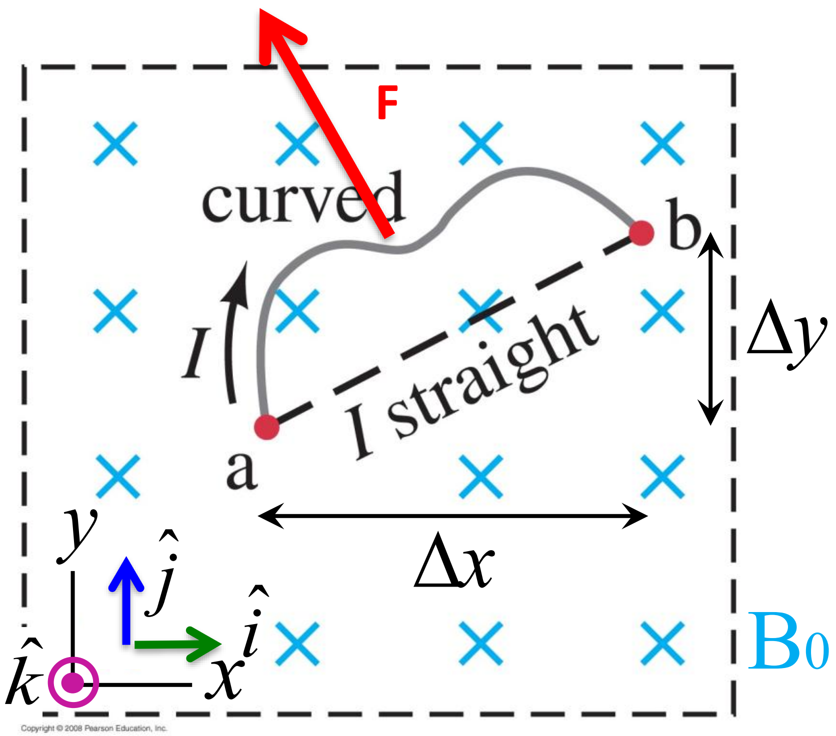
$$\mathbf{F}_B = I\mathbf{L}' \times \mathbf{B}$$



The magnetic force on a curved current-carrying wire in a uniform magnetic field is equal to that on a straight wire connecting the end points and carrying the same current.

Magnetic force to a current in wire

A curved wire, connecting two points **a** and **b**, lies in a plane perpendicular to a uniform magnetic field B0 and carries a current *I*.



$$\left|\vec{\mathbf{F}}\right| = IB_0\left|\left(-\Delta y\hat{\mathbf{i}} + \Delta x\hat{\mathbf{j}}\right)\right|$$

$$\left|\left(-\Delta y\hat{\mathbf{i}} + \Delta x\hat{\mathbf{j}}\right)\right| = \sqrt{\Delta y^2 + \Delta x^2} = l$$

l is the length of the stray segment from **a** to **b**

$$\left|\vec{\mathbf{F}}\right| \equiv F = IB_0 l$$

$$\vec{\mathbf{F}} = \int_a^b I\,d\vec{\mathbf{L}} \times \vec{\mathbf{B}} = I\int_a^b \left(\hat{\mathbf{i}}dx + \hat{\mathbf{j}}dy\right) \times \left(-B_0\hat{\mathbf{k}}\right) = IB_0\int_a^b \left(\hat{\mathbf{j}}dx - \hat{\mathbf{i}}dy\right) =$$

$$\hat{\imath} \times \hat{k} = -\hat{\jmath} \qquad \hat{\jmath} \times \hat{k} = \hat{\imath}$$

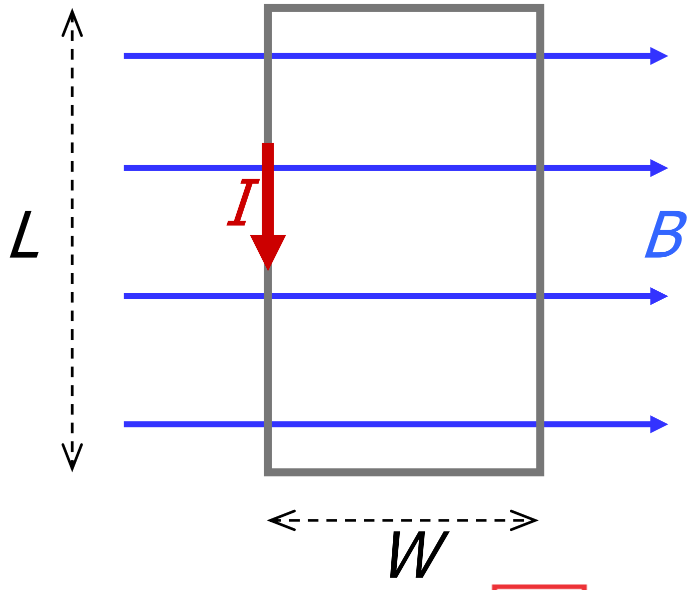
$$\vec{\mathbf{F}} = IB_0\left(-\Delta y\hat{\mathbf{i}} + \Delta x\hat{\mathbf{j}}\right)$$

The resultant magnetic force on the wire, no matter what its shape, is the same as that on a straight wire connecting the two points carrying the same current I.

Magnetic Forces and Torques on Current Loops

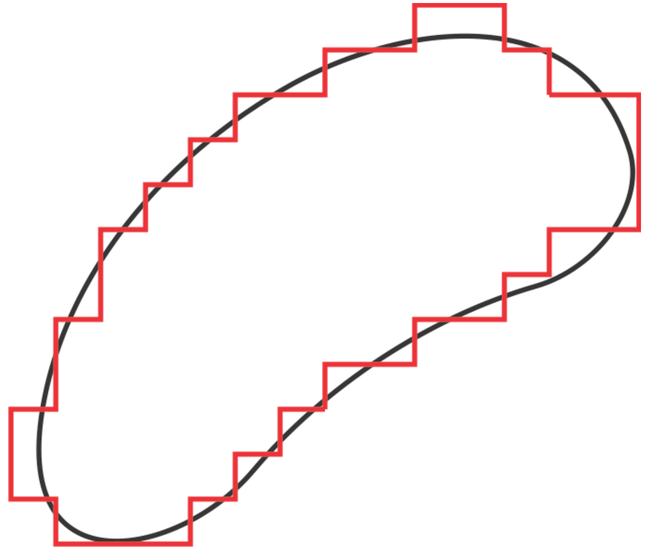
Rectangular loop:

- *currents in left and right wires are equal and opposite*
- *magnetic forces on left and right wires are equal and opposite*
- *same for top and bottom wires*
    - ***net magnetic force on loop is zero***



General shape loop:

- *can be decomposed into rectangular pieces*



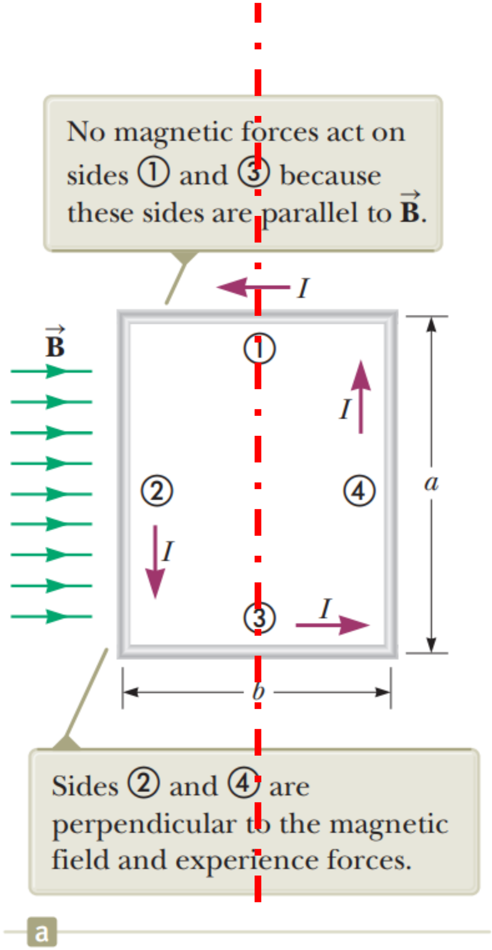
Net force on any closed current loop is zero

Torque on a Current Loop in a Uniform Magnetic Field

Consider a rectangular loop carrying a current I in the presence of a uniform magnetic field directed parallel to the plane of the loop as shown in Figure.

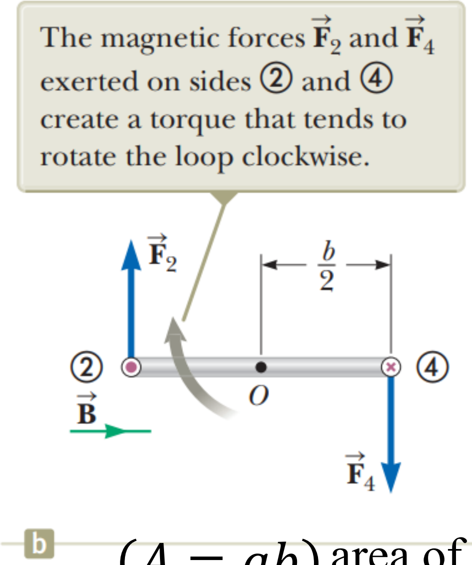
No magnetic forces act on sides (1) and (2) because these wires are parallel to the field

A non-zero magnetic forces acts on sides (2) and (4) because these sides are oriented perpendicular to the field.



A zero total net force on the loop!

$$F_2 = F_4 = IaB$$



$(A = ab)$ area of the loop

$$\tau_{max} = IAB$$

F_2 and F_4 point in opposite directions but are not directed along the same line of action. If the loop is pivoted so that it can rotate about point O, **these two forces produce about O a torque that rotates the loop clockwise**

$$\tau_{max} = F_2 \frac{b}{2} + F_4 \frac{b}{2} = (IaB)\frac{b}{2} + (IaB)\frac{b}{2} = IabB$$

- This maximum-torque result is valid only when the magnetic field is parallel to the plane of the loop. The sense of the rotation is clockwise.
- If the current direction were reversed, the force directions would also reverse, and the rotational tendency would be counterclockwise.

Torque on a Current Loop in a Uniform Magnetic Field

Now suppose that the uniform magnetic field makes an angle >90° with a line perpendicular to the plane of the loop.

The magnetic forces F1 and F3 exerted on sides 1 and 3 cancel each other and produce no torque because they pass through a common origin.

However, the magnetic forces F2 and F4 acting on sides 2 and 4 produce a torque about any point.

$$\tau = F_2 \frac{b}{2}\sin\theta + F_4 \frac{b}{2}\sin\theta$$
$$= IaB\left(\frac{b}{2}\sin\theta\right) + IaB\left(\frac{b}{2}\sin\theta\right)$$
$$= IAB\sin\theta$$

$(A = ab)$ area of the loop

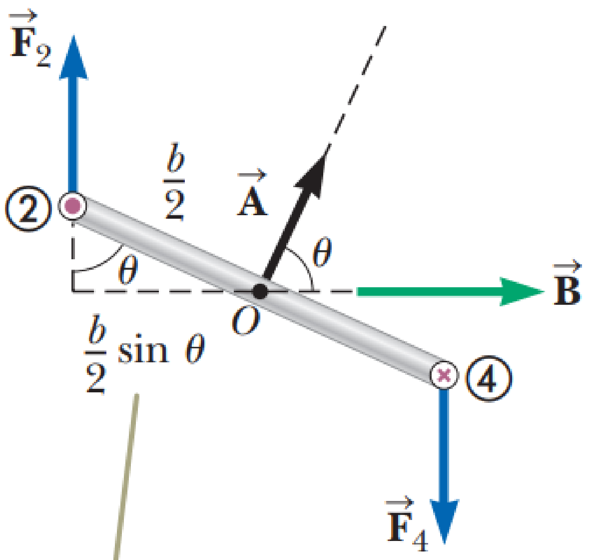
A convenient expression for the torque exerted on a loop placed in a uniform magnetic field:

$$\vec{\tau} = I\vec{A} \times \vec{B}$$

magnetic dipole moment

$$\vec{\mu} \equiv I\vec{A}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

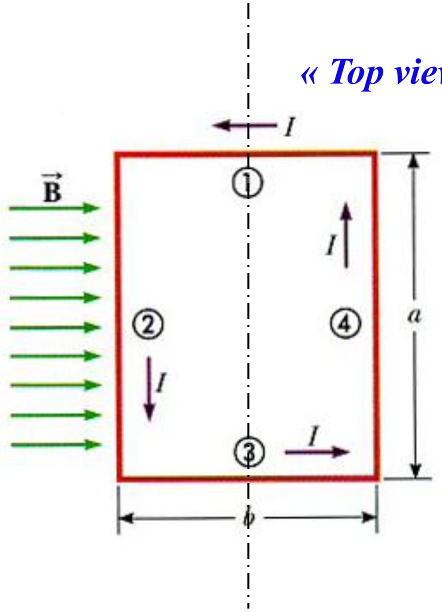


When the normal to the loop makes an angle θ with the magnetic field, the moment arm for the torque is $(b/2)\sin\theta$.

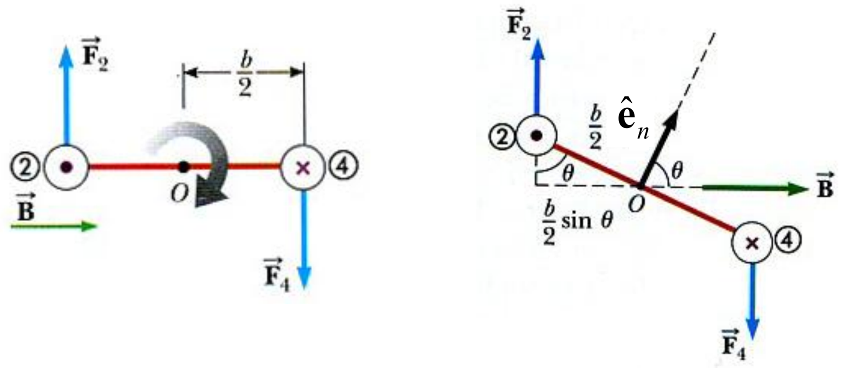
The torque has its maximum value IAB when the field is perpendicular to the normal to the plane of the loop (θ = 90) *and is zero when the field is parallel to the normal to the plane of the loop* (θ = 0) .

Summary: Force and torque
in a circuit in a uniform magnetic field

« Top view »



« Lateral view »



$$\mathbf{F}_i = I\mathbf{L}_i \times \mathbf{B} \quad \Rightarrow \quad F_2 = F_4 = IaB\,; \quad \mathbf{F}_1 = \mathbf{F}_3 = 0$$

$$\boldsymbol{\tau}_i = \mathbf{r}_i \times \mathbf{F}_i \quad \Rightarrow \quad \tau = bIaB\sin\theta = IAB\sin\theta\;; \quad \boldsymbol{A} = ab\hat{\mathbf{e}}_n = A\hat{\mathbf{e}}_n$$

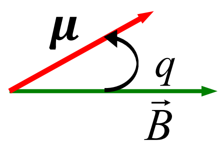
$\Rightarrow$

$$\boldsymbol{\tau} = I\boldsymbol{A} \times \mathbf{B} = \boldsymbol{\mu} \times \mathbf{B}$$

Torque on a circuit (of any shape and orientation)

$$\boldsymbol{\mu} \triangleq I\boldsymbol{A}$$

"**Magnetic dipole**" of the circuit



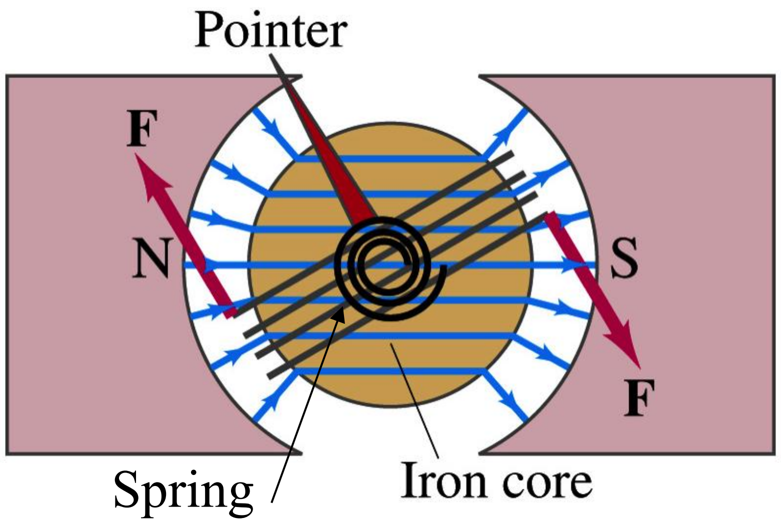
$d\theta > 0 \rightarrow dW < 0$

$$dW = -\tau\, d\theta = -|\boldsymbol{\mu}|B\sin\theta\, d\theta = -dU_B$$

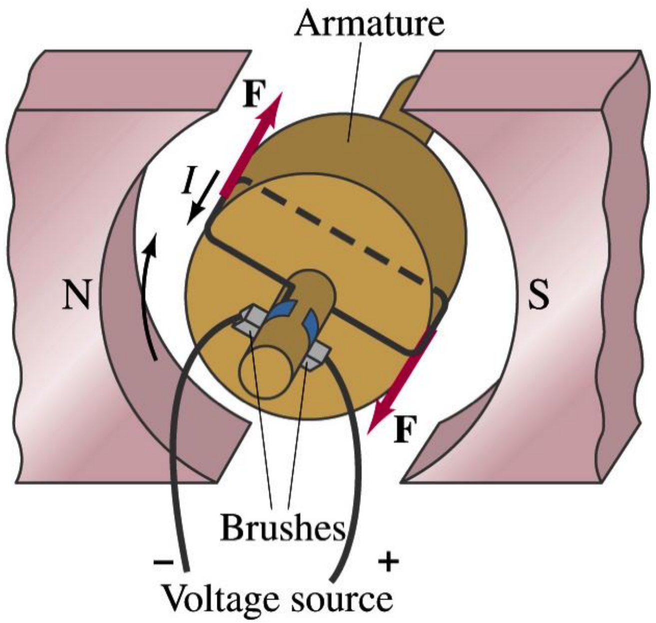
$$U_B = -|\boldsymbol{\mu}|B\cos\theta = -\boldsymbol{\mu}\cdot\mathbf{B}$$

Magnetic potential energy

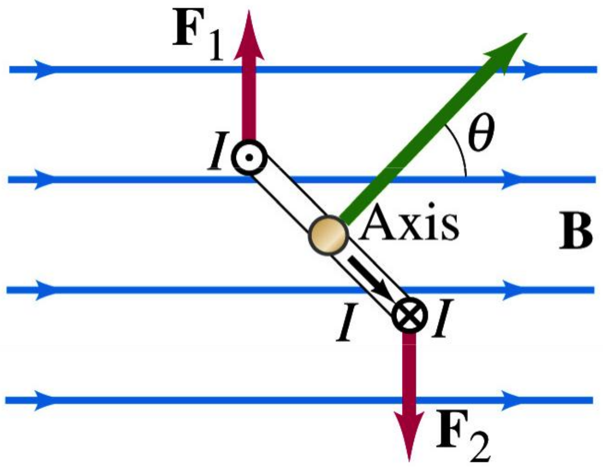
Examples of "torque on a loop" applications



Galvanometer
(electric current measurement)



Motor



The D'Arsonval Galvanometer

A D'Arsonval galvanometer is shown in Figure. When the turns of wire making up the coil carry a current, the magnetic field created by the magnet exerts on the coil a torque that turns it (along with its attached pointer) against the spring. The angle of deflection of the pointer is directly proportional to the current in the coil.

We can use Equation $\boldsymbol{\tau} = \boldsymbol{\mu} \times \boldsymbol{B}$ to find the torque τ_m that the magnetic field exerts on the coil.

If we assume that the magnetic field through the coil is perpendicular to the normal to the plane of the coil, we have

$$\tau_m = \mu B$$

This magnetic torque is opposed by the torque due to the spring, which is given by the rotational version of Hooke's law,)

$$\tau_s = -\kappa\phi$$

where κ, is the torsional spring constant and ϕ is the angle through which the spring turns.

Because the coil does not have an angular acceleration when the pointer is at rest, the sum of these torques must be zero:

$$\tau_m + \tau_s = \mu B - \kappa\phi = 0$$

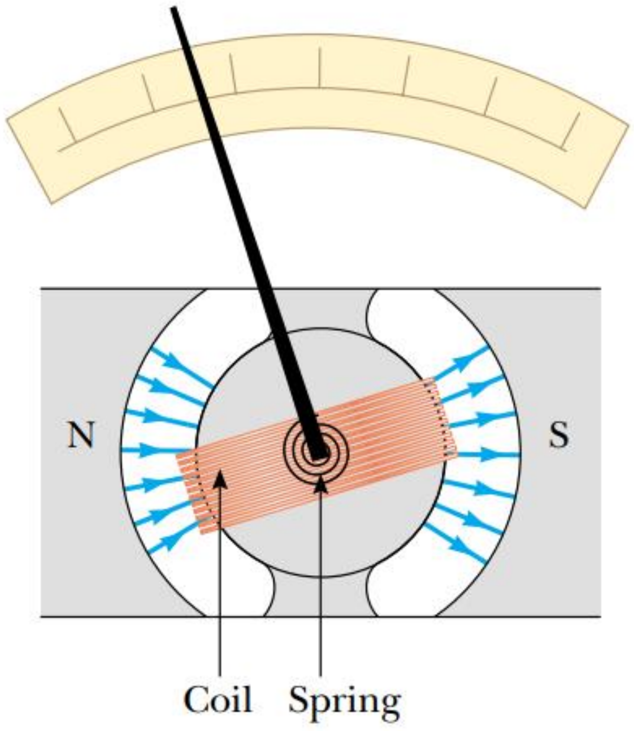
The magnetic moment of the N turns of wire to the current through them: $\mu = NIA$

$$(NIA)B - \kappa\phi = 0$$

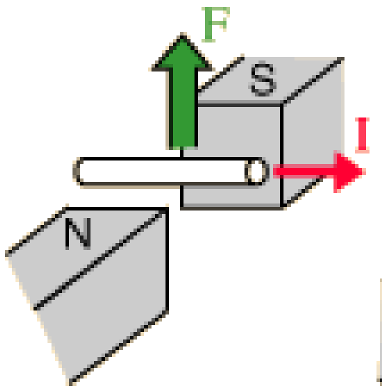
Thus, the angle of deflection of the pointer is directly proportional to the current in the loop. The factor NAB/κ tells us that deflection also depends on the design of the meter.

The circular cross section of the magnet ensures radial magnetic field lines.

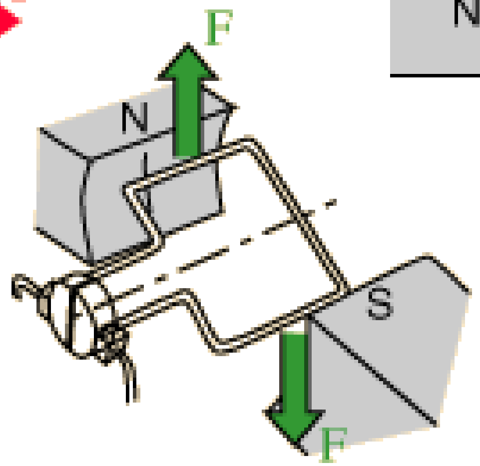
$$\phi = \frac{NAB}{\kappa} I$$



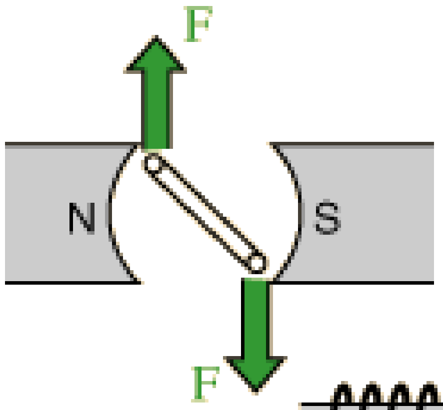
Electric Motors



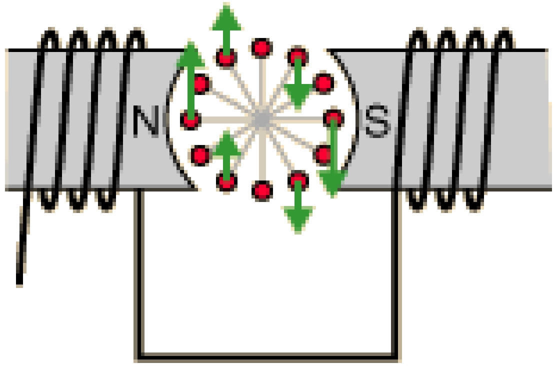
An electric current in a magnetic field will experience a force.



If the current-carrying wire is bent into a loop, then the two sides of the loop which are at right angles to the magnetic field will experience forces in opposite directions.



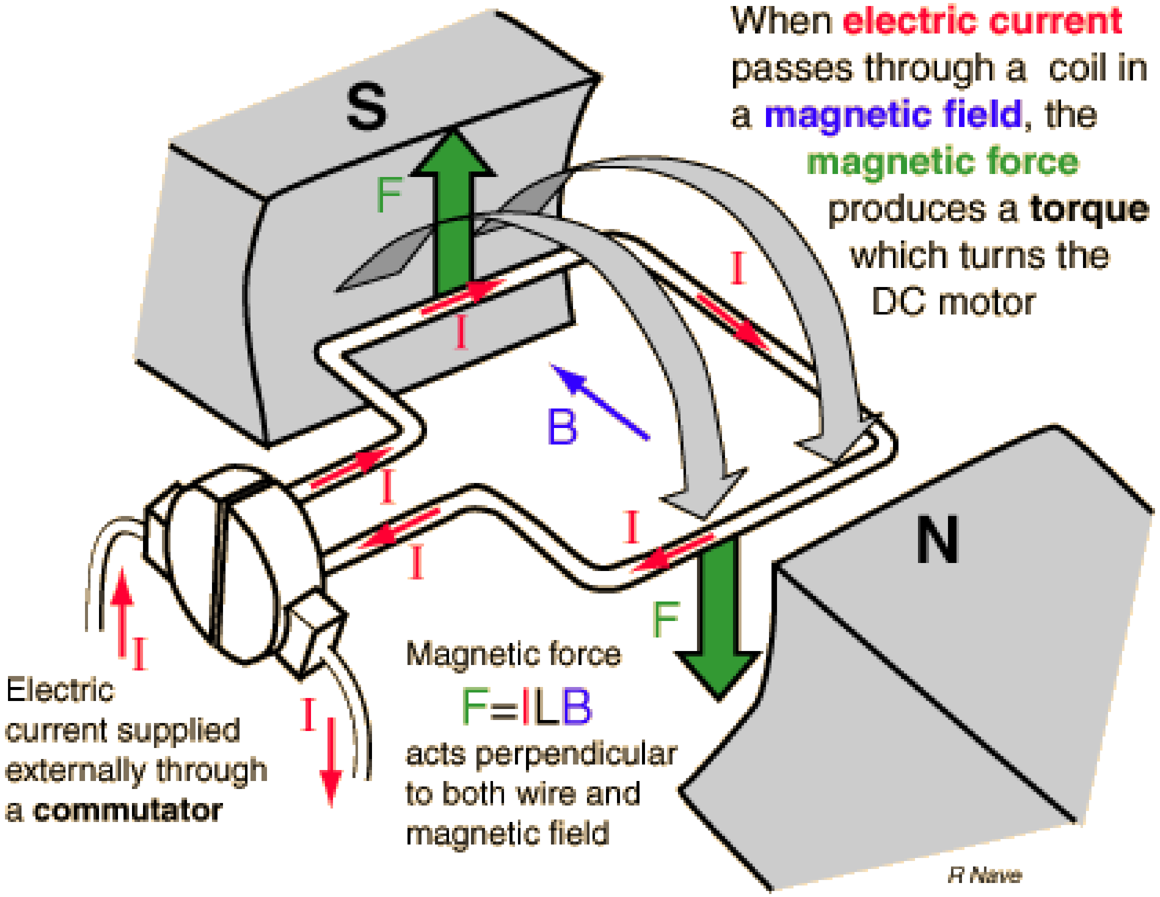
The pair of forces creates a turning influence or torque to rotate the coil.

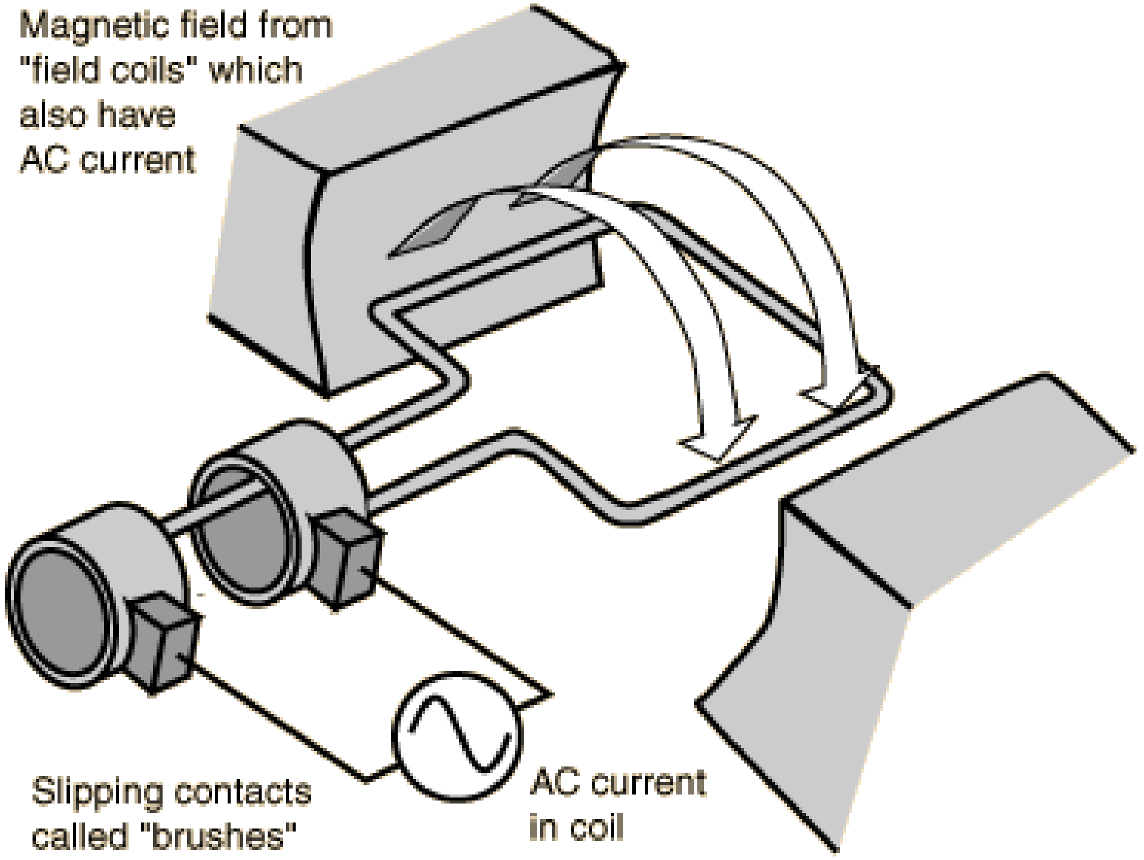


Practical motors have several loops on an armature to provide a more uniform torque and the magnetic field is produced by an electromagnet arrangement called the field coils.

Electric Motors

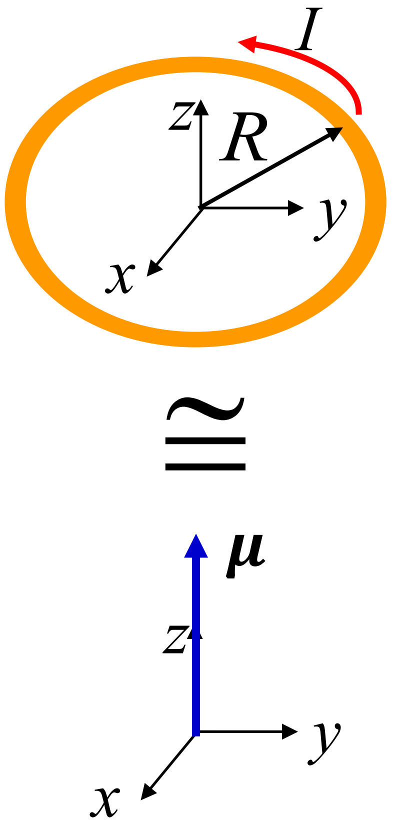
graphics showing how dc and ac motors work.





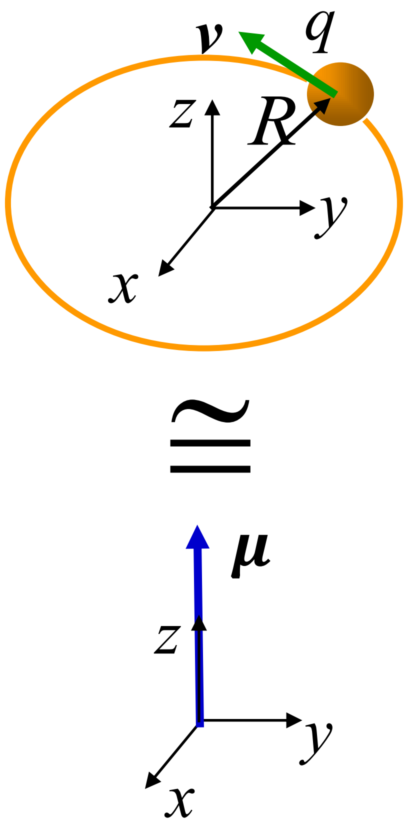
Models of the “magnetic dipole”

Ampere model
(current in circular turn)



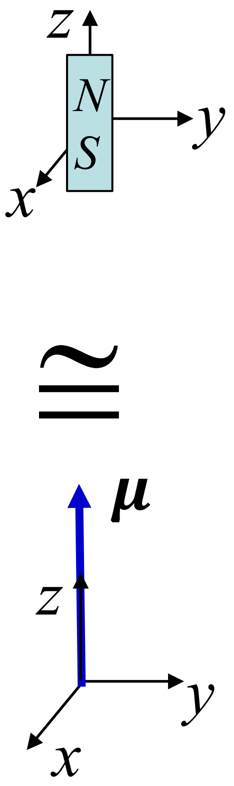
$\boldsymbol{\mu} = I\pi R^2 \hat{\mathbf{z}}$

(moving charge
on a circular path)



$\boldsymbol{\mu} = q\dfrac{v}{2}R\hat{\mathbf{z}}$

Gilbert's model
(magnet)

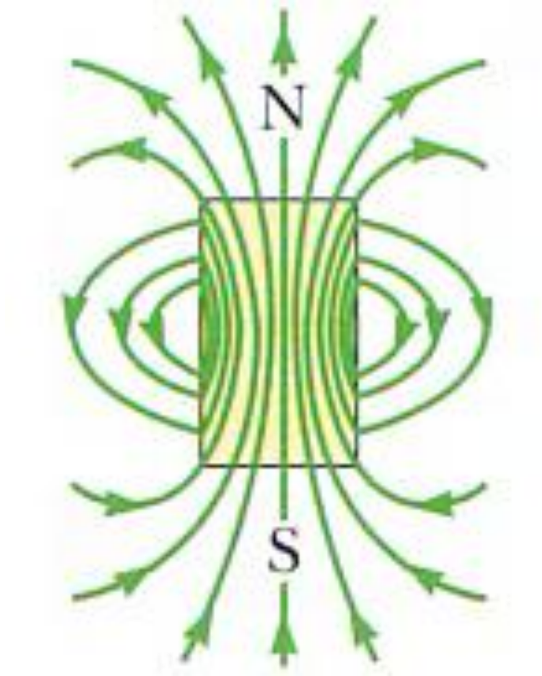


$\boldsymbol{\mu} = MV\hat{\mathbf{z}}$

Ampère's principle of equivalence

between a small magnet and a current loop

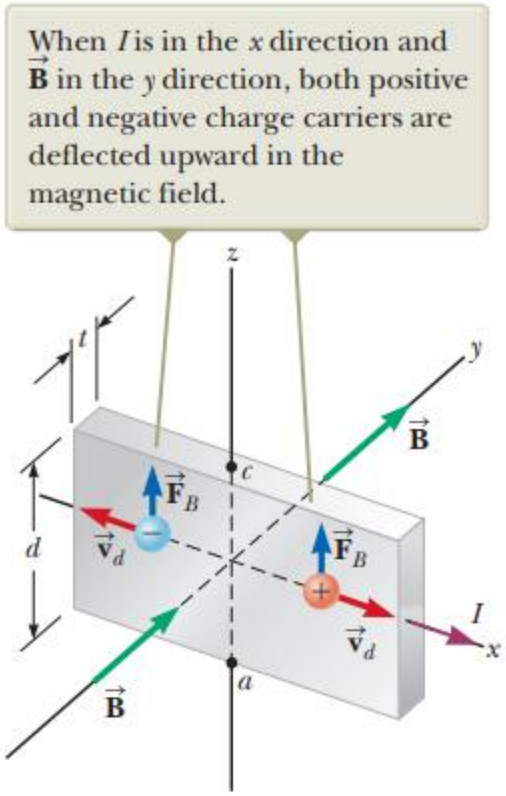
$\mathbf{m} = I\pi R^2 \hat{\mathbf{z}} = IA\hat{\mathbf{z}}$ [Am^2]
R: loop radius [m]
I: current [A]
A: loop area [m^2]



$\mathbf{m} = MV\hat{\mathbf{z}}$ [Am^2]
$\mathbf{M}$: magnetization of the magnet [A/m]
(magnetic dipole density)
V: volume of the magnet [m^3]

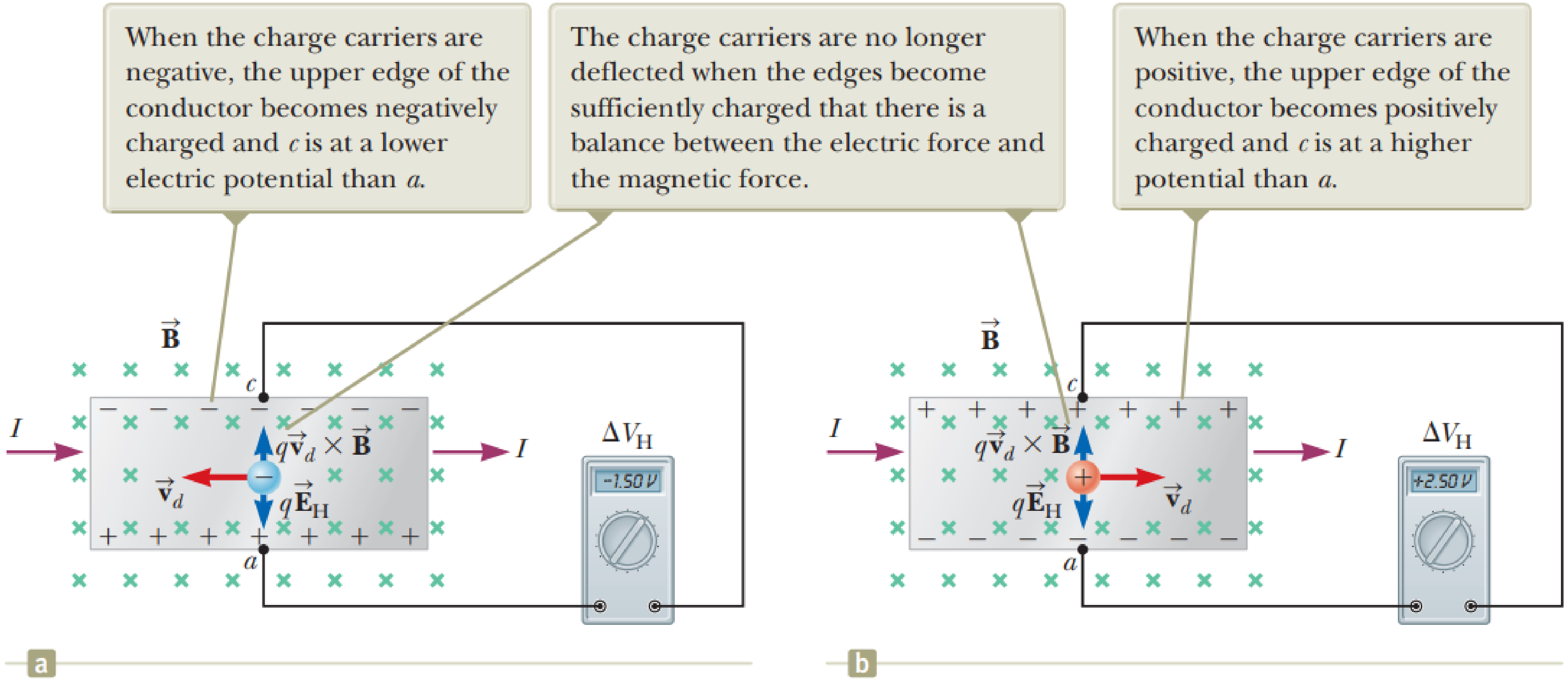
1. Same behaviour in an external magnetic field (rotation, displacement)
2. Same field **B** created (at a great distance)

The Hall Effect



When a current-carrying conductor is placed in a magnetic field, a potential difference is generated in a direction perpendicular to both the current and the magnetic field.

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}$$



$$qv_dB = qE_H$$

$$E_H = v_dB$$

If d is the width of the conductor, the Hall voltage is

$$\Delta V_H = E_Hd = v_dBd$$